

How to Motivate Fundamental Innovation: Subsidies versus Prizes and the Role of Venture Capital

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Abstract

In this paper, we analyze the mechanisms through which governments and non-profit agencies (principals) may incentivize the development of fundamental innovations, defined as those innovations that have positive social value net of development costs, but have negative net present values to the innovating firms due to the limited appropriability of their value by such firms. We solve for the principal's optimal choice between subsidy schemes and incentivizing prizes under two alternative assumptions: first, where the producing firms' innovation development costs are observable by the principal; and second, where such costs are unobservable. We then introduce venture capitalists (VCs) into the above setting and analyze how VCs may enhance the efficiency of the principal in motivating fundamental innovations. Our analysis has several implications for how to better incentivize innovation development, and in particular, for how government-funded venture capitalists may enhance innovation.

JEL classification: G32, G38.

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1 Introduction

The objective of this paper is to analyze “fundamental innovation.” By fundamental innovation we mean innovation projects which involve large fixed costs by way of investment for implementation, but for which the benefits are spread out across various sectors of society so that the value of these benefits are not easily appropriable by a single firm. This implies that individual firms will be reluctant to undertake such projects, since given the large upfront costs and the limited appropriability of the value of the benefit from successful implementation, such projects will be evaluated as negative net present value projects from the point of view of any given firm. Of course, the notion that innovative activities are difficult to finance in a freely competitive marketplace has been around for a long time, perhaps starting with the classic articles of Nelson (1959) and Arrow (1962).¹ For example, Nelson (1959) talks about the difficulty of financing basic research in a competitive marketplace and the desirability of government subsidies. However, the problem of underinvestment in innovative activities may not be confined to basic science. In fact, even in the presence of various mechanisms to increase appropriability such as patents or other forms of intellectual property protection, some of the most important innovative projects from the point of view of social value (e.g., developing a more efficient battery to store energy) may suffer from underinvestment because of high development costs and limited appropriability of the value of their benefits.^{2,3}

¹The idea itself was first alluded to by Schumpeter (1942).

²A recent book that argues that many of the fundamental innovations underlying such popular products as the iPod and iPhone manufactured by Apple, and Google’s search algorithm, were funded by the U.S. government in various ways (e.g. through agencies like DARPA or the National Science Foundation) is Mazzucato (2015). This book also lists many of the innovations funded by governments around the world, and argues that government funding is at the heart of motivating fundamental innovation, directly leading to the establishment of many industries, such as the solar power and those fostering green technologies in general. While the points made by this book are undoubtedly true, it is also true that a large proportion of government funding in various countries aimed at promoting innovation is wasted or skimmed off by corrupt bureaucrats or entrepreneurs. The objective of this paper is to establish the most efficient way for the government to incentivize the private sector to develop innovations when such help is warranted.

³Of course, the problem of underinvestment in worthwhile projects with great social value but low private value goes beyond the area of innovation development. One of the most famous examples of the difficulty in financing such a project is that of the difficulty faced by Christopher Columbus in funding his expedition to discover a sea route to India (later America), referred to as the “Enterprise of the Indies.” While he was ultimately successful in obtaining financing from Queen Isabella of Spain, his request for financing was turned down by King John II of Portugal, Henry VII of England, and Charles VIII of France. In addition to high upfront investment costs and limited appropriability, this project also shared with innovation development another important feature: high uncertainty

The traditional way in which society has addressed such underinvestment in fundamental innovation are either subsidies by governments or non-profit foundations, or incentivizing prizes, again provided by the government or non-profit foundations. For example, the U.S. Department of Defense often heavily subsidizes firms that are shortlisted to participate in its design competitions (see, e.g., Lichtenberg (1988) and Che and Gale (2003)). Thus, the department allocates a significant portion of its R&D budget to be awarded to small businesses undertaking military research. The award of prizes to solve important technological hurdles also has a long and illustrious history. In 1598, King Philip III of Spain, having lost many of his ships returning from the West Indies, offered a monetary prize for the first person to discover a method for measuring longitude (Leerberg (2008)). Following up on this, the British Parliament offered a prize of 20,000 pounds for a “practical and useful” means of determining longitude at sea (Che and Gale, 2003). In 1829, the investors of the Liverpool and Manchester Railway set aside a winner-take-all prize of 500 pounds for the design of the most improved locomotive engine (Day, 1971). More recently, the U.S. Department of Defense has sponsored design competitions to stimulate research into technology useful for department uses: an example is the 2005 DARPA (Defense Advanced Research Projects Agency) Grand Challenge, which is a 132.2 mile long race among autonomous robots in the Mojave Desert. There has been a recent resurgence in the use of prizes for spurring innovation in areas considered to be socially and economically important, not only by the government, but by non-profit foundations and even private companies: see, e.g., Brunt, Lerner, and Nicholas (2011) for some recent examples.⁴ Finally, the government has recently started programs where they channel some subsidies through venture capital funds to increase investment in certain targeted sectors.

The objective of this paper is to analyze fundamental innovations in the sense described above

about the probability of success.

⁴The following are some recent prizes: Brunt, Lerner, and Nicholas (2011) note that the X-Prize Foundation awarded a \$10 million prize for suborbital spaceflight in 2004, followed by a \$10 million prize for rapid human genome sequencing, the \$30 million Google moon challenge (a competition to land a privately funded robot on the moon), and inducements for clean-tech and medical related solutions. NASA has sponsored prizes for technological innovation since 2004 and several other governmental prize challenges, or advance market commitments, have been announced (Kalil, 2006). They further note that a pioneering venture fund, Prize Capital, has sought to use contests to generate investment opportunities.

and study how investment in such innovations can be increased through various mechanisms by the government and by private agents such as venture capitalists. We consider a setting where there is a producer sector, which consists of firms capable of developing an innovation and having a continuum of development costs for the innovation. If the innovation is successfully developed, however, firms in the producer sector will be able to appropriate only a fraction of the overall value of the innovation. The remaining fraction of the value of the innovation will go to firms in a user sector. The larger the proportion of firms in the producer sector attempting to develop the innovation, the greater the probability of successful development. We first show that there will be underinvestment by the private sector in developing the innovation relative to the socially optimal value.

We then analyze how the government can motivate an increase in the fraction of firms in the producer sector attempting to develop the innovation toward the socially optimal value. We analyze two possible mechanisms that may be used by the government to motivate innovation by the private sector: subsidies or grants to specific firms for undertaking the development of a certain innovation or alternatively, an “innovation prize” to be awarded to firms that contributed to the development of a successful innovation. We compare the efficacy of these two mechanisms for motivating innovation in two alternative settings: first, when the innovation development costs are observable to the government and second, when such costs are not observable. Finally, we extend our model to analyze the role of financial intermediaries such as venture capitalists in motivating investment in fundamental innovations by the private sector. Here we study how the presence of such intermediaries affects the extent of government subsidies required to motivate an increase in private sector investment in developing the innovation to the socially optimal level.

We develop a number of interesting new results in the above setting. We first characterize the situation in which there is no government intervention and the private sector decides to invest in fundamental innovations based on the profit motive: i.e., based on their calculation of the net present value each firm is able to generate from the innovation. We show that, in this case, there

will be underinvestment in innovation, since only the lowest-cost firms (for which the innovation is positive NPV, despite the fixed cost of developing the innovation and the lack of complete appropriability of the value generated by the innovation) will invest in attempting to develop the innovation. We also show that, in this setting, the underinvestment in innovation will be greatest in those cases where the upfront costs of developing the innovation are the greatest and the appropriability of the innovation by the producer is the lowest (conversely, the fraction of value going to the user industry is the greatest). This result is important, since it shows that the extent of underinvestment may be greatest in some fundamental innovations that may bring very large benefits to society as a whole: e.g., vaccines against certain widespread diseases; or revolutionary technologies, which, while extremely costly to develop, may completely revolutionize many industries in addition to the producer industry.

We then turn to the case where the government (or equivalently, a non-profit foundation) may intervene to increase investment in innovation by the private sector, either through providing subsidies or through providing an incentivizing prize, in the event of developing a successful innovation, to those firms that invested in developing such an innovation. We first analyze the case where the cost structure of firms attempting to develop the innovation is known to the government. In this setting, there are two important differences between subsidies and prizes. First, since the costs of various firms are observable by the government, it can provide targeted subsidies to various individual firms (depending on their cost structure), unlike in the case of a prize, where the award of the prize is the same for all firms. Second, since subsidies to firms are provided *ex ante* and based on individual firm costs, there is room for “skimming” a fraction of the amount provided, unlike in the case of a prize, where a flat prize is provided only in the event of success to all firms that invested in developing the innovation. We first show that, regardless of whether the government intervention comes in the form of a subsidy or a prize, the government is unable to achieve the first best level of innovation if there are frictions like the deadweight cost of raising the amount required for government intervention (through subsidies or a prize) from taxes. Further, we show that, as

long as the fraction of subsidies “skimmed” by individual firms is small, subsidies always dominate awarding an incentivizing prize in terms of motivating innovation. This is because, when the cost structure of innovating firms is observable, the resources saved from the ability to provide targeted subsidies dominates any resources lost through skimming by individual firms (provided that the latter fraction is not too large). This result implies that, in the case of innovations where the cost structure for developing the innovation and the leading players are known to the government, providing targeted subsidies will generate a greater amount of innovation relative to providing a prize for the successful development of the innovation.

We then analyze and develop results for the situation where the cost structure of innovating firms in the relevant industry is not observable by the government, and the government may not even be aware of the relevant firms that can successfully undertake the development of the innovation. In this case, the advantage of subsidies over prizes, namely, the ability to target certain firms and provide them with the “right level” of subsidies (corresponding to their individual development costs) is no longer feasible. In this case, if the government attempts to provide subsidies upfront, it is likely that they may end up subsidizing many firms that are incapable of developing the innovation, thus wasting at least a fraction of the subsidy provided to such firms (even assuming that the government can induce such firms to regurgitate the bulk of the subsidy provided). A prize, on the other hand, is provided *ex post*, and only to firms that invested in developing a successful innovation, so that the lack of observability up front of the cost structure or even the identity of firms that can develop the innovation is not a disadvantage. Thus, even when skimming by firms receiving subsidies is small, subsidies are a very inefficient way of motivating innovation in this case, and offering a flat prize dominates subsidies for a wide range of parameter values: i.e., for the same resources spent by the government, it can motivate a larger fraction of firms in the producer sector to invest in developing innovations. However, even in the case of a prize, the first best optimum is never achieved given a positive deadweight cost of raising the resources used to award a prize

through taxation.⁵

Finally, we develop results in a setting where venture capitalists are present in the economy and can take equity positions in firms that may invest in developing innovations. We show that, even in the absence of government intervention, venture capitalists are able to spur innovation to higher levels than in their absence. The intuition here is that venture capitalists are able to take significant equity positions in firms both in the sector producing innovations and also in the user sector, consisting of firms that can benefit from using the innovation in the future. This means that venture capitalists are able to internalize some of the benefits from the innovation going to the user sector as well as the producer sector, so that an innovation project that has a negative NPV from the point of view of an entrepreneur in the producer sector may turn out to have positive NPV from a venture capitalist's point of view. This, in turn, implies that venture capitalists have an incentive to motivate some entrepreneurs (who would otherwise not invest) to invest in developing an innovation by providing them with value transfers (e.g., in the form of cheap equity financing). We then analyze the case with both VC financing and government intervention, and show that, when the venture capitalists are present in the economy, the government is able to induce the same level of innovation as in their absence, but with a smaller expenditure of resources in the form of subsidies. We conduct this analysis under two alternative settings: in the first setting, both the government and VCs can observe the innovation development costs of firms; in the second setting, only VCs can observe these innovation development costs. Our analysis in the second setting above provides a theoretical rationale for government-funded venture capital investments.

The rest of the paper is organized as follows. Section 2 discusses how our paper is related

⁵The advantage of prizes over subsidies when the principal (government agency or non-profit foundation) does not know where the innovation may come from, seems to be recognized in practice. As Leerberg (2008) points out, this seems to be the motivation underlying NASA's "Centennial Challenges" designed to stimulate innovation and competition in solar system exploration and ongoing NASA mission areas. Further, in describing the initiative, NASA notes that the awards are based on "actual achievements, instead of proposals," and seek solutions "from non-traditional sources of innovation." Brant Sponberg, the program manager for the project, explained that NASA used the "Centennial Challenges" prizes as a mechanism to "reach out to sources of innovation wherever they might lie." (see the article by Michael A. Prospero, "Fuel for Thought," FAST COMPANY, Jan/Feb 2006). In a similar vein, John Harrison, who won the 20,000 pound prize offered by the English government for developing a sufficiently accurate chronometer, was also an amateur (Leerberg (2008)).

to the existing literature. Section 3 presents our basic model. Section 4 builds on our basic model to analyze the case of observable costs with government intervention. Section 5 analyzes the case of unobservable costs with government intervention. Section 6 extends our basic model to analyze the role of venture capital in fostering innovation and how this interacts with government intervention. Section 7 describes the testable predictions and policy implications of our model. Section 8 concludes. The proofs of all propositions are confined to the Appendix.

2 Relation to the Existing Literature

Our paper is related to three different strands in the literature. The first strand is the literature on the role of governments (and non-profit foundations) in motivating innovations and the mechanisms through which they may do so. It suggests that prize awards can be a powerful mechanism for accelerating technological development (e.g., Polanyi (1944), Wright (1983), Kremer (1998), Shavell and Ypersele (2001), Scotchmer (2004), Boldrin and Levine (2008), Kremer and Williams (2009), Chari et al., 2009)). Fu, Lu, and Lu (2012) study how a principal attempts to minimize the amount of time it takes to discover an innovative technology by allocating a fixed budget between a prize and subsidies. Taylor (1995), Fullerton and McAfee (1999), and Che and Gale (2003) examine how a buyer of a technology should design the contract offered to competing firms in order to increase the quality of their output. Moldovanu, Sela, and Shi (2010) investigate the optimal contest design when the designer can reward high performing agents with positive prizes and low performing agents with negative prizes in order to maximize participants' efforts. Fernandez, Stein, and Lo (2013) propose megafunds that invest in a well-diversified portfolio of research programs to reduce the risk of innovation. It is worth noting that none of the above papers have analyzed the trade-offs between prizes and subsidies in motivating innovation that we study in this paper. Further, unlike our paper, none of the above studies here explore the role of financial intermediaries (such as venture capitalists) in motivating innovation and the role of interactions between venture

capitalists and the government in motivating innovation and thereby enhancing social welfare.

The second strand in the literature our paper is related to is the theoretical literature on managing and motivating innovation: see, e.g., Holmstrom (1989), Aghion and Tirole (1994), or Manso (2011). The third strand of literature our paper is related to is the empirical literature on the role of venture capital in motivating innovation. Tian and Wang (2011) find that IPO firms backed by more failure-tolerant VC investors are significantly more innovative and VC failure tolerance is particularly important for ventures that are subject to high failure risk. Lerner (2012) suggests that perhaps the best way to motivate innovation is a “hybrid” model, such as a corporate venture capital (CVC) program, that combines features of corporate research laboratories and venture-backed start-ups “within a powerful system that consistently and efficiently produces new ideas.” Chemmanur, Loutskina, and Tian (2014) empirically compare the efficacy of independent and corporate venture capitalists in motivating innovation.

The fourth and final strand in the literature our paper is also related to the empirical literature on the role of government venture capital in motivating innovation. Alperovych, Hübner, and Lobet (2015) investigate the implications of venture capital (VC) investor type (government or private) on the operating efficiency of a sample of Belgian portfolio firms up to 3 years after the investment. Brander, Du, and Hellmann (2015) find that enterprises funded by both government venture capitalists (GVCs) and private venture capitalists (PVCs) obtain more investment than enterprises funded purely by PVCs, and much more than those funded purely by GVCs.

3 The Basic Model

Consider an economy with two industries: firms in industry P are the providers of a certain innovation (“the producer industry”), and firms in industry U are the end users of the same innovation (“the user industry”). If the innovation is successfully developed, firms in the user industry U will make use of this innovation to collectively create a positive value of $v_0 > 0$. However, as we will

discuss later, firms in the producer industry P are able to capture only a fraction of this value. If the innovation is not successfully developed, then industry U will create zero value (since they have no innovation to use). There is a continuum of firms in industry P. We normalize the total number of firms in industry P to 1 and assume that they are uniformly distributed over the interval $[0, 1]$. We index the firms in industry P by their investment cost of developing the innovation. That is, the cost of developing the innovation for each firm $i \in [0, 1]$ is represented by a cost density function that is equal to iC , where C is a positive constant.

The probability that the innovation is successfully developed by firms in industry P depends on the number (fraction) of firms that invest in developing the innovation. For tractability, we assume that this probability is characterized by $\mu \times p$, where $\mu \in [0, 1]$ is the Lebesgue measure of the subset of firms in industry P that incur the investment cost to try to develop the innovation, and $p \in (0, 1)$ is an exogenous constant. We can interpret p as the maximum probability that the innovation will be developed successfully in case all firms in industry P invest in developing the innovation. We assume that $pv_0/C < 1$ to avoid corner solutions.

If the innovation is successfully developed by firms in industry P, these firms will capture only a fraction $\alpha \in (0, 1)$ of the total value created by firms in industry U: i.e., the innovating firms will capture a total value of αv_0 , which will be shared among them uniformly. Since innovating firms have a total Lebesgue measure of μ in industry P, the value captured by each innovating firm has a uniform density of $\frac{\alpha v_0}{\mu}$. If the innovation is not successfully developed, firms in industry P will capture a value of 0.

Broadly, the objective of the principal (government or non-profit funding agency) is to maximize the total social value created by the innovation minus any costs involved in the development of the innovation (incurred by both the principal and firms in industry P). The principal has to make the following decisions: a) leave the private sector to develop the innovation and not intervene, or b) to intervene either by providing subsidies or offering a prize to promote investments in innovation by firms in industry P. If a subsidy scheme is used, the principal will offer a gross subsidy of S_i to

each firm i . We assume that a fraction γ of the gross subsidy amount S_i is skimmed by some firm employees so that the net amount of subsidy that covers some portion of firm i 's cost of developing the innovation is $(1-\gamma)S_i$.⁶ If a prize is offered, innovating firms in industry P (with a total measure of μ) will share a prize of Z when the development is successful. Otherwise, they will receive a payoff of 0.

We assume that the principal faces a shadow cost of public funds $\delta > 0$. Thus, each dollar spent by the principal on a subsidy scheme or a prize offering is raised through distortionary taxes (labor, capital, and excise taxes) and costs society $(1 + \delta)$ dollars. Laffont and Tirole (1993) indicate that the measurement of the shadow cost of public funds, for example, results from the theory of public finance and from the estimation of the elasticities of demand and supply for consumption, labor, and capital. According to several studies (see Ballard, Shoven, and Whalley (1985), Hausman and Poterba (1987)), the average shadow cost of public funds for the U.S. economy is estimated to be $\delta = 0.3$ or 30%. The social cost of public funds due to deadweight losses of taxation is likely to be higher in countries with less efficient tax collection.

Definition of Equilibrium: In sections 3 to 6, we define equilibrium as the solution to the principal's (government or non-profit foundation) problem, subject to the participation and incentive compatibility constraints of the innovating firms. In section 6, where we have some strategic interaction between the government and the venture capitalist (VC), we continue to define equilibrium as the solution to the principal's problem, subject to the participation and incentive compatibility constraints of the innovating firms, with the additional requirement that the equilibrium behaviors specified also constitute a perfect Bayesian equilibrium (PBE) of the game between the principal and the venture capitalist.

⁶The assumption of skimming is meant to reflect the fact that, in many situations, when accounting, legal, and other control systems are not perfect, there is an incentive for firms and their employees to misallocate (divert) government subsidies and to extract private benefits from them, possibly because the monitoring of the use of such funds is weaker than that of private funds belonging to these firms. Such skimming is likely to be greater in less developed economies with less efficient accounting and legal systems. note that we allow the skimming parameter γ to vary between 0 and 1, so that our results are not driven exclusively by this skimming parameter: we also derive results for the case where γ is very small, so that the skimming involved is negligible.

3.1 The Benchmark Case

We first analyze the benchmark case in which the principal as a social planner determines the first-best allocation of innovation development among firms in industry P in the absence of any frictions like externalities and costs of intervention. The amount of innovation development that maximizes the social surplus in this case is given by solving the following problem:

$$\max_{\mu} \quad \mu p v_0 - \int_0^{\mu} i C di. \quad (1)$$

In this objective function, the first term is the expected value of the innovation for the entire economy and the second term is the aggregate cost of innovation development. In other words, the principal chooses the optimal fraction μ of innovating firms in industry P to maximize the social NPV from innovation development.

Proposition 1 (*The Benchmark Case of Innovation Development*) *The first best fraction of firms developing the innovation is given by:*

$$\mu^* = \frac{p v_0}{C}. \quad (2)$$

The total social surplus under this first best solution is given by:

$$U^* = \frac{p^2 v_0^2}{2C}. \quad (3)$$

This proposition shows that the first best level of innovation development μ^* in industry P is increasing in the value v_0 created by the innovation and the highest possible value of the probability of success p . On the other hand, the higher the cost parameter C , the greater the aggregate cost of research and development, and therefore the lower the first best level of innovation development in the economy.

3.2 The Private Sector Equilibrium without Intervention

If the principal does not intervene, the fraction of firms in industry P that have an incentive to develop the innovation by themselves may differ from that in the benchmark case above, since firms in industry P can capture only a fraction α of the total value v_0 created by the innovation. In a competitive private sector equilibrium, an individual firm will invest in developing the innovation only if it is privately optimal for it to do so regardless of the social value of the innovation. The next proposition characterizes the fraction of firms trying to develop the innovation without any intervention in equilibrium.

Proposition 2 (*Equilibrium without Intervention from the Principal*) *Suppose the principal does not intervene in industry P.*

(i) *The fraction of firms that invest in developing the innovation, μ_0 , is characterized by*

$$\mu_0 = \frac{\alpha p v_0}{C}. \quad (4)$$

Firms $i \in [0, \frac{\alpha p v_0}{C}]$ will invest in developing the innovation, while firms $i \in (\frac{\alpha p v_0}{C}, 1]$ will not invest in developing the innovation. The total social surplus is given by:

$$U_0 = \frac{(2\alpha - \alpha^2)p^2 v_0^2}{2C}. \quad (5)$$

(ii) *When $\alpha < 1$, there is underinvestment in developing the innovation in the competitive equilibrium.*

The intuition underlying the above proposition is as follows. Suppose a fraction $\mu \in (0, 1)$ of firms in industry P invest in developing the innovation. Then, the payoff to each firm is as follows. With probability μp , the innovation is successful, and the payoff to each innovating firm i is $\frac{\alpha v_0}{\mu}$. Otherwise, its payoff is zero. Thus, the expected gross payoff for a firm that invests in developing the innovation is

$$\mu p \frac{\alpha v_0}{\mu} + (1 - \mu p)0 = \alpha p v_0. \quad (6)$$

At the same time, the cost of developing the innovation is iC for firm i , so each firm i will privately decide whether or not to invest in the innovation project based on the difference between its

expected payoff and its cost. Firm i will invest in developing the innovation if and only if the following condition holds:

$$\alpha p v_0 - i C \geq 0. \tag{7}$$

In equilibrium, for the marginal firm i_0 that invests in developing the innovation, the condition given in (7) is satisfied as an equality so that $i_0 = \mu_0 = \frac{\alpha p v_0}{C}$. Given the threshold μ_0 and the decision rule in (7), firms $i \in [0, \frac{\alpha p v_0}{C}]$ will implement the project, while firms $i \in (\frac{\alpha p v_0}{C}, 1]$ will not implement the project.

Note that the competitive private sector equilibrium achieves the first best level of innovation development of the benchmark case if and only if $\alpha = 1$. If $\alpha < 1$, there is underinvestment in innovation in the competitive equilibrium, since $\mu_0 = \frac{\alpha p v_0}{C} < \mu^* = \frac{p v_0}{C}$. In the next few sections, we will analyze how the principal can improve upon the competitive equilibrium outcome by motivating a larger number of firms to invest in developing the innovation, thereby increasing the total surplus created by the innovation activities of the firms in industry P.

4 The Case of Government Intervention with Observable Development Costs

In this section, we assume that the principal can observe the cost structure of individual firms. This means that the government can observe the cost index i for any individual firm $i \in [0, 1]$ in industry P. We will consider two intervention mechanisms that the government can use to motivate firms to invest in developing an innovation: subsidies and prizes.

4.1 The Equilibrium with Subsidies when Development Costs are Observable

We first consider the provision of subsidies by the principal to motivate innovation development across a larger number of firms compared to the competitive private sector equilibrium. We know that firms with $i > \mu_0$ will not invest in implementing the innovation unless the principal intervenes

to provide an incentive to them to do so, since the condition for innovation development to be privately optimal for them does not hold for them:

$$\alpha v_0 p - iC < 0, \quad \forall i \in (\mu_0, 1]. \quad (8)$$

Suppose the principal provides a subsidy of S_i to motivate a fraction μ of firms to try to develop the innovation, where $\mu > \mu_0$ (we solve for the optimal μ below). In particular, the principal conjectures that it is optimal to subsidize only some firms in the continuous interval $[0, \mu]$, i.e., that the optimal value of μ is less than 1 so that firms in the interval $(\mu, 1]$ will not receive a subsidy, i.e., $S_i = 0$ for all $i \in (\mu, 1]$.

Given that a measure μ of firms invest in developing the innovation, the innovation is successfully developed with probability μp , and the payoff to each firm in the case of success is $\frac{\alpha v_0}{\mu}$. Since a fraction γ of the subsidy S_i is skimmed by firm employees, the expected payoff to an innovation-developing firm ($i \in [0, \mu]$) is

$$\mu p \frac{\alpha v_0}{\mu} + (1 - \mu p)0 - iC + (1 - \gamma)S_i = \alpha v_0 p - iC + (1 - \gamma)S_i. \quad (9)$$

In this setting, the principal's objective function is given by:

$$\max_{\mu, S_i} (1 - \alpha)\mu p v_0 + \int_0^\mu (\alpha v_0 p - iC + (1 - \gamma)S_i) di - \int_0^\mu (1 + \delta)S_i di \quad (10)$$

$$\text{s.t. } \alpha v_0 p - iC + (1 - \gamma)S_i \geq 0 \quad \forall i \in [0, \mu]. \quad (11)$$

The first term in this objective function is the expected externality of the innovation enjoyed by firms in the user industry U. The second term represents the expected payoff of firms in the producer industry in the range $[0, \mu]$, which invest in developing the innovation. Note that, as can be seen from the principal's objective function (10), the principal is maximizing only that part of

the subsidy which goes toward covering the shortfall between costs and expected cash flows for these firms, i.e., $(1 - \gamma)S_i$, and not the part which is skimmed by the employees of these subsidized firms, i.e., γS_i . Finally, the third term is the cost of the subsidy to the principal (the government) and the tax payers. The constraint given in (11) is the individual rationality constraint for a firm $i \in [0, \mu]$ to invest in developing the innovation.

Proposition 3 (*Equilibrium with Subsidies in the Case of Observable Costs*) *Suppose the principal can observe the cost structure of individual firms, and it provides subsidies to incentivize firms in industry P to invest in innovation development:*

(i) *The fraction of firms that invest in developing the innovation, μ_{so}^* , is characterized by*

$$\mu_{so}^* = \left[\alpha + \frac{(1 - \gamma)}{(1 + \delta)}(1 - \alpha) \right] \frac{pv_0}{C}. \quad (12)$$

Firms $i \in [0, \mu_0]$ will invest in developing the innovation without receiving a subsidy. Firms $i \in (\mu_0, \mu_{so}^]$ will invest in developing the innovation by receiving a subsidy*

$$S_i^* = \frac{iC - \alpha v_0 p}{1 - \gamma}, \quad (13)$$

while firms $i \in (\mu_{so}^, 1]$ will not invest in developing the innovation and receive a zero payoff.*

(ii) *The total social surplus in this case is given by:*

$$U_{so} = \frac{\left(2\alpha - \alpha^2 + \frac{(1 - \gamma)}{(1 + \delta)}(1 - \alpha)^2 \right) p^2 v_0^2}{2C}. \quad (14)$$

The principal prefers to subsidize firms in industry P rather than not intervene, since $U_{so} > U_0$.

(iii) *The equilibrium amount of innovation development μ_{so}^* is decreasing in the subsidy-skimming percentage, γ , and decreasing in the shadow cost of public funds due to distortionary taxation, δ .*

The above proposition shows that, in the case of observable costs, a targeted subsidy scheme can indeed help the principal motivate a larger number of firms to invest in developing the innovation compared to the case of no intervention, i.e., $\mu_{so}^* > \mu_0$. This is largely due to the fact that the principal also includes the expected payoff $(1 - \alpha)v_0 p \mu$ captured by the user industry U into his objective function. However, the provision of subsidies is also costly to the principal who must take into account the deadweight costs associated with raising taxes to fund the subsidies and the cost

of skimming by employees of the innovating firms that occurs during the transfer of subsidies to the innovating firms. Therefore, the principal's objective is to maximize the incentives of firms in industry P to generate innovation while subsidizing them in the most cost-efficient way.

In the competitive equilibrium of the private sector, we showed that firms in the interval $[0, \mu_0]$ have an incentive to try to develop the innovation without any subsidies as their expected payoff from innovation exceed their investments costs, i.e., $\alpha v_0 p - iC \geq 0$ for all $i \in [0, \mu_0]$. Given that the principal can observe the costs of these firms, he finds it optimal not to subsidize them so that $S_i^* = 0$ for all $i \in [0, \mu_0]$. Similarly, the principal does not want to incentivize any firm with an innovation development cost greater than $\mu_{so}^* C$, as the marginal social cost of subsidizing these firms is greater than the marginal social benefit of having them invest in developing the innovation. Therefore, the principal optimally sets $S_i^* = 0$ for all firms in the interval $(\mu_{so}^*, 1]$ as well.

For all firms in the range $i \in (\mu_0, \mu_{so}^*]$, we know that their innovation development cost exceeds their expected gross payoff from the innovation, i.e., $iC > \alpha v_0 p$. However, the principal finds it optimal to offer a positive net subsidy that covers this shortfall between their development costs and expected cash flows, since the marginal social benefit of motivating these firms to invest in developing the innovation exceeds the marginal social cost of their innovation developmet. Further, it is optimal to set $S_i^* = \frac{iC - \alpha v_0 p}{1 - \gamma}$ for all $i \in (\mu_0, \mu_{so}^*]$, i.e., the constraint given in (10) is binding in equilibrium, since the principal prefers to minimize the total cost of the provision of subsidies to maximize the total surplus.

One should also note that the optimal level of innovation μ_{so}^* in this case is only a second best compared to the first best level μ^* in the benchmark case, i.e., $\mu_{so}^* < \mu^*$, since the principal cannot avoid incurring some deadweight costs associated with raising tax funds for the subsidies ($\delta > 0$) and a positive fraction ($\gamma > 0$) of the subsidy S_i^* to each firm $i \in (\mu_0, \mu_{so}^*]$ is skimmed off in the process.

4.2 The Equilibrium with a Prize when Development Costs are Observable

When the principal decides to use a prize mechanism to motivate innovation by firms in industry P, the prize is offered *ex post* only to firms that invested in developing the innovation and collectively succeeded in developing the innovation.⁷ We assume that the principal can observe *ex post* which firms in industry P invested in developing the innovation or not. Given that the prize offered is Z and the fraction of firms that invest in developing the innovation is μ , the expected value of the total prize offered by the principal is $\mu p Z$.⁸ Further, if the innovation is developed successfully, the prize amount Z will be uniformly distributed among all innovation-developing firms in interval $[0, \mu]$ so that the prize payoff of such a firm is $\frac{Z}{\mu}$. Therefore, the expected prize payoff to an atomistic firm in $[0, \mu]$ is

$$\mu p \frac{Z}{\mu} + (1 - \mu p) 0 = pZ, \quad (15)$$

and the expected net payoff density of firms in the interval $[0, \mu]$ must satisfy the following participation constraint:

$$\alpha p v_0 - iC + pZ \geq 0. \quad (16)$$

Firms in the interval $(\mu, 1]$ will not invest in developing the innovation, since the following inequality will hold:

$$\alpha p v_0 - iC + pZ < 0. \quad (17)$$

⁷Note that in practice, prizes are offered only to firms which both invested in developing the innovation and succeeded in developing the innovation individually. In our model, we have a continuum of (uncountable) firms for mathematical tractability. Therefore, we assume that an innovation is developed successfully (with probability μp) as the outcome of the collective efforts of a fraction of μ firms that invested in developing the innovation, the prize is uniformly shared by all such firms. In Appendix B, we show that, in expected value terms, this is equivalent to the prize going only to those firms that individually succeeded in developing the innovation independently of each other. In expected value terms, it is also similar to the case in which the prize goes to the firm that develops the innovation first, since, as more firms try to develop the innovation, the probability of any given firm being the first one to succeed goes down.

⁸When the principal can observe the cost structure of individual firms, it is theoretically possible to design a prize mechanism that is conditional on the cost index i of an individual firm. However, in practice, such mechanisms are not observed, and a typical prize specifies a fixed reward of Z in the case of a successful innovation development. Therefore, we focus on the typical flat (unconditional) prize mechanism in our theoretical analysis.

The objective function of the principal in this case is then given by:

$$\max_{\mu, Z} (1 - \alpha)\mu pv_0 + \int_0^\mu (\alpha pv_0 - iC + pZ)di - \int_0^\mu (1 + \delta)pZdi \quad (18)$$

$$\text{s.t. } \alpha pv_0 - iC + pZ \geq 0 \quad \forall i \in [0, \mu], \quad (19)$$

and the participation constraint given in (17) for all firms with $i \in (\mu, 1]$. The first term in this objective function is the expected externality of the innovation enjoyed by firms in the user industry U. The second term represents the expected payoff of firms in the range $[0, \mu]$, which invest in developing the innovation. Finally, the third term is the cost of the prize to the principal (the government) and tax payers. The principal chooses the amount of the prize Z and the fraction of firms μ that it wants to incentivize in industry P to develop the innovation in order to maximize the total social surplus in (18), subject to the participation constraint (19).

Proposition 4 (*Equilibrium with a Prize when Costs are Observable*) *Suppose the principal offers a prize Z to incentivize firms in industry P to invest in innovation development:*

(i) *The fraction of firms that invest in developing the innovation, μ_z^* , is characterized by*

$$\mu_z^* = \frac{(1 + \alpha\delta) pv_0}{(1 + 2\delta) C}. \quad (20)$$

Firms $i \in [0, \mu_z^]$ will invest in developing the innovation. They will be candidates to win the prize Z^* , which is given by:*

$$Z^* = \frac{(1 - \alpha(1 + \delta))}{(1 + 2\delta)} v_0, \quad (21)$$

while firms $i \in (\mu_z^, 1]$ will not invest in developing the innovation and receive a zero payoff.*

(ii) *The total social surplus in this case is given by:*

$$U_{zo} = \frac{(1 + \alpha\delta)^2 p^2 v_0^2}{(1 + 2\delta) 2C}. \quad (22)$$

The principal prefers to award a prize rather than not intervene, i.e., $\mu_z^ > \mu_0$, if and only if $\alpha(1 + \delta) < 1$.*

(iii) *The equilibrium amount of innovation development μ_z^* is decreasing in the shadow cost of public funds due to distortionary taxation, δ , and increasing in the fraction of value, α , that firms in industry P capture from successful innovation development. The optimal amount of the prize offered Z^* is decreasing in δ and decreasing in α .*

The above proposition shows that, if the social cost of public funds δ is low and the fraction α of the value of the innovation that can be appropriated by the innovating firms in industry P is not too large, offering a prize also helps the principal motivate a larger number of firms to invest in developing the innovation compared to the case of no intervention, i.e., $\mu_z^* > \mu_0$ if and only if $\alpha(1 + \delta) < 1$.⁹

The equilibrium prize amount Z^* given in (21) is determined by the break-even condition of the marginal firm whose cost index is equal to μ_z^* . This means that the participation constraint given in (19) is satisfied as an equality for the marginal firm with $i = \mu_z^*$, so that the following condition holds:

$$Z^* = \frac{\mu_z^* C}{p} - \alpha v_0. \quad (23)$$

The principal does not want to offer a prize to any firm with an investment cost greater than $\mu_z^* C$, since the marginal social cost of offering a prize to these firms is greater than the marginal social benefit of having them invest in developing the innovation. Therefore, in equilibrium, firms in the interval $(\mu_z^*, 1]$ will not try to develop the innovation.

Note that since a prize is offered *ex post* here, the principal does not suffer from skimming costs (γ) unlike in the case of subsidies, which are offered to firms *ex ante*. However, a prize is awarded to all firms which contributed to the development of a successful innovation, and the greater the social cost of public funds δ associated with raising distortionary taxes, the more expensive it becomes for the principal to use and fund a prize to motivate firms to innovate.¹⁰ Therefore, the fraction μ_z^* of firms that invest in developing the innovation and the optimal amount of the prize Z^* are both decreasing in δ . Nevertheless, if the positive externality benefits of a greater amount of innovation are significantly large (i.e., α is not too high) and the shadow cost of public funds δ is not too high, a prize mechanism helps the principal to achieve a higher total surplus by motivating a larger number

⁹Note also that this condition is equivalent to $\delta < \frac{1-\alpha}{\alpha}$ as well as $\alpha < \frac{1}{1+\delta}$.

¹⁰Note that in the extreme case in which raising taxes entails no deadweight costs, i.e., when $\delta = 0$, the principal is able to implement the benchmark first best outcome by offering a prize $Z^* = (1 - \alpha)v_0$. In this case, firms in industry P would be able to increase their total expected payoff by the entire positive externality of the innovation through the prize.

of firms to innovate. The optimal amount of the prize Z^* given in (21) is decreasing in α . This is because, the greater the appropriability of the value of the innovation by firms in the producer industry P, i.e., the higher is α , the smaller is the firms' need for the principal's costly provision of high-powered incentives (by way of the prize amount) to invest in developing the innovation.

4.3 The Comparison of Subsidies versus Prizes when Development Costs are Observable

Next, we analyze the principal's optimal choice between subsidies versus prizes in the case of observable innovation development costs. The principal's objective is to incentivize firms' investment in developing the innovation by choosing the most efficient intervention mechanism available.

Proposition 5 (*The Choice between Subsidies and Prizes when Costs are Observable*)
Let $\alpha(1 + \delta) < 1$. In motivating firms in industry P to invest in innovation development when development costs are observable, the principal's choice between providing subsidies and offering a prize is characterized as follows:

- (i) If the skimming parameter γ is low so that $0 \leq \gamma \leq \bar{\gamma}_1$, the principal prefers subsidizing firms in industry P to offering a prize to them. In this case, the fraction of firms that invest in developing the innovation when the principal uses a subsidy scheme is greater than that when he uses a prize offering, i.e., $\mu_{so}^* > \mu_z^*$. The threshold value of the skimming parameter $\bar{\gamma}_1$ is given by:

$$\bar{\gamma}_1 = \frac{\delta(1 + \alpha\delta)}{(1 - \alpha)(1 + 2\delta)}. \quad (24)$$

- (ii) If the skimming parameter γ is moderately high so that $\bar{\gamma}_1 < \gamma \leq \bar{\gamma}_2$, the principal prefers subsidizing firms in industry P to offering a prize to them. In this case, the fraction of firms that invest in developing the innovation when the principal uses a subsidy scheme is smaller than that when he uses a prize offering, i.e., $\mu_{so}^* < \mu_z^*$. The threshold value of the skimming parameter $\bar{\gamma}_2$ is given by:

$$\bar{\gamma}_2 = \frac{\delta(1 + 2\alpha\delta - \alpha^2(1 + \delta(3 + \delta)))}{(1 - \alpha)^2(1 + 2\delta)}. \quad (25)$$

- (iii) If the skimming parameter γ is very high so that $\gamma > \bar{\gamma}_2$, the principal prefers offering a prize to firms in industry P to subsidizing them. In this case, the fraction of firms that invest in developing the innovation when the principal uses a prize offering is greater than that when he uses a subsidy scheme, i.e., $\mu_z^* > \mu_{so}^*$.

This proposition shows that unless the skimming parameter γ is substantially high, i.e., $\gamma > \bar{\gamma}_2$, the provision of subsidies dominates the offering of a prize as an intervention mechanism for the

principal in the case of observable innovation development costs. The intuition underlying this result is as follows. While subsidies can be targeted to specific firms in the industry depending on their investment cost iC , this cannot be done in the case of a prize. Note that, in the case of subsidies analyzed above, the principal does not provide any subsidies to firms with $i < \mu_0$, since these firms would have invested in developing the innovation without the government's intervention anyway. Similarly, for firms in the interval $(\mu_0, \mu_{so}^*]$, the subsidy provided to each firm i is a function of its investment cost iC , so that the firm just breaks even in investing in R&D after receiving its subsidy. On the other hand, when a prize is offered, the prize is set at a relatively high (and uniform) level so that only the marginal firm with cost index μ_z^* breaks even, whereas all other firms in the interval $[0, \mu_z^*)$, which have lower investment costs, earn positive rents. This implies that, given a positive social cost of public funds $\delta > 0$, in many cases (i.e., as long as $\gamma < \bar{\gamma}_2$), the principal optimally chooses to subsidize firms rather than to offer a prize, since it is more cost-effective for the principal to encourage firms to innovate using subsidies.

Consistent with the above intuition, we also show that, when the skimming cost of providing a subsidy is sufficiently low, i.e., if $\gamma < \bar{\gamma}_1$, the principal can induce a larger fraction of firms to innovate using targeted subsidies at a low cost to the government compared to the case in which he uses a prize offering: $\mu_{so}^* > \mu_z^*$. However, if the skimming cost is moderately high, i.e., if $\bar{\gamma}_1 < \gamma \leq \bar{\gamma}_2$, we show that the government can induce only a slightly lower fraction of firms to innovate using subsidies compared to the fraction of firms that it could have induced to innovate using a prize offering: $\mu_{so}^* < \mu_z^*$.¹¹ Nevertheless, the principal chooses to incentivize firms to innovate using subsidies, since, if the skimming of subsidies is not too large, the cost advantage of providing targeted subsidies only to a subset of innovating firms still dominates the inefficiency of awarding the same level of prize to all innovating firms (when the principal uses a prize mechanism).

¹¹Numerical simulations of our model show that, in this intermediate region in which $\bar{\gamma}_1 < \gamma \leq \bar{\gamma}_2$, the difference in the fractions of firms investing to develop the innovation, $\mu_z^* - \mu_{so}^*$, when the principal uses a prize mechanism versus a subsidy scheme, is very small even though it is positive.

The only case when prize dominates subsidy when costs are observable is when the economy is very corrupt (poor accounting systems, low ability to monitor) so that the skimming parameter γ is very large, i.e., $\gamma > \bar{\gamma}_2$. In this case, the advantage of a targeted subsidy scheme over a prize mechanism is entirely swamped by the high cost of skimming that occurs in the process of subsidizing firms. Further, since $\gamma \gg \bar{\gamma}_1$ in this case, the principal is able to induce a significantly larger fraction of firms in industry P to innovate using a prize mechanism rather than a subsidy scheme, tilting the principal's choice even further in the direction of a prize mechanism.

5 The Case of Government Intervention with Unobservable Development Costs

In this section, we assume that the principal cannot observe the innovation development cost structures of individual firms. Thus, the cost index i of any individual firm $i \in [0, 1]$ in industry P is unobservable to the principal. However, we still assume that the principal can observe *ex post* which firms in industry P invested in developing the innovation and which did not. We will again compare subsidies and prizes in terms of their cost-effectiveness in motivating firms to invest in developing an innovation.

5.1 The Equilibrium with Subsidies when Development Costs are Unobservable

We now consider the provision of subsidies when the principal cannot observe the cost structure of individual firms. In this case, incentive compatibility requires that the principal has to offer the same amount of subsidy, S , to all firms in industry P.^{12, 13} For a given level of subsidy S , the

¹²In other words, we can show that in any direct mechanism that asks firm i to report its type (cost index) truthfully in our setting of unobservable costs, the subsidy S_i transferred from the government to firm i must be constant for all $i \in [0, 1]$ provided that the principal cannot contract on firms' R&D investment policies. Otherwise, if there exist two firms with types i and j , respectively, where $i \neq j$ and $S_j > S_i$, then firm i has an incentive to report its type as j rather than i as long as the principal has no control over firm i 's R&D investment policy.

¹³It is possible that, while the government is unable to directly observe the innovation development costs of firms, it may be able to rule out certain firms in an industry as incapable of developing the innovation. In this case, we include in the set of firms in industry P that the government subsidizes only those firms that it assesses as being capable of developing the innovation.

principal knows that there exists a critical fraction μ of firms that will invest in developing the innovation, while the rest of the firms, $(1 - \mu)$ will not invest in developing the innovation. As in the case of observable costs, the principal realizes that every firm that receives the subsidy will skim a fraction γ of the subsidy. Further, in this setting with unobservable innovation development costs, there is an incentive for firms that have prohibitively high innovation development costs to apply and receive subsidies as long as they can retain or divert at least a fraction of this subsidy amount in the form of private benefits. We capture this incentive by assuming that even firms that do not find it worthwhile to invest in developing the innovation due to their high costs even after receiving the subsidy S may nevertheless apply for and obtain subsidies, and retain a fraction λ of this subsidy amount $(1 - \gamma)S$ before returning the rest to the principal.^{14, 15}

For firms that invest in developing the innovation, i.e., for $i \in [0, \mu]$, the following incentive compatibility constraint must hold:

$$\alpha p v_0 - iC + (1 - \gamma)S \geq \lambda(1 - \gamma)S, \quad (26)$$

so that they invest iC in developing the innovation after receiving a net subsidy $(1 - \gamma)S$ rather than retain a fraction λ of it and return the rest to the principal without developing the innovation. For firms in the interval $(\mu, 1]$, the principal also has to offer the same subsidy S to each firm, but they will not attempt to develop the innovation, but will instead divert a fraction λ of the subsidy as private benefits, and return the rest to the principal, since the following inequality holds for these firms:

$$\alpha p v_0 - iC + (1 - \gamma)S < \lambda(1 - \gamma)S. \quad (27)$$

¹⁴This assumption may also be motivated (outside our model) by the possibility that firms in industry P may not even know their own cost structure and may spend part of the subsidy to figure out if it is worthwhile for them to fully implement the research development project (by using the full amount of the subsidy S and invest iC).

¹⁵Clearly, the principal cannot avoid giving subsidies to such firms with prohibitively high innovation development costs, since he does not observe firms' cost structure. In principle, the principal may be able to force such firms to return the lion's share of the subsidies *ex post* when it becomes clear that they have not invested in developing the innovation. However, it is unlikely that the entire subsidy amount is returned to the principal, and at least a fraction may be wasted or diverted by the recipient firm as private benefits.

The objective function of the principal in this case is then given by:

$$\max_{\mu, S} (1 - \alpha)\mu pv_0 + \int_0^\mu (\alpha pv_0 - iC + \bar{S}) di - (1 + \delta) \left(\int_0^\mu S di + \int_\mu^1 (1 - (1 - \lambda)(1 - \gamma)) S di \right) \quad (28)$$

$$\text{s.t. } \alpha pv_0 - iC + (1 - \lambda)(1 - \gamma)S \geq 0 \quad \forall i \in [0, \mu], \quad (29)$$

$$\alpha pv_0 - iC + (1 - \lambda)(1 - \gamma)S < 0 \quad \forall i \in (\mu, 1]. \quad (30)$$

where $\bar{S} \equiv (1 - \lambda)(1 - \gamma)S$. The first term in the objective function is the expected externality of the innovation. The second term represents a major part of the expected payoff of firms that invest in developing the innovation. Note again that the principal wants to maximize only that part of the subsidy, which goes toward covering the shortfall between costs and expected cash flows for these innovating firms, i.e., $(1 - \lambda)(1 - \gamma)S$.¹⁶ The third term represents the principal's cost of subsidizing firms that invest in developing the innovation ($i \in [0, \mu]$) and the cost of the amount wasted on subsidizing firms that do not invest in developing the innovation even after receiving a subsidy ($i \in (\mu, 1)$). The constraints in (29) and (30) are obtained after rearranging the IC constraints in (26) and (27), respectively.

Proposition 6 (*Equilibrium with Subsidies in the Case of Unobservable Costs*) *Suppose the principal cannot observe the cost structure of individual firms, and it provides subsidies to incentivize firms in industry P to invest in innovation development:*

(i) *The fraction of firms that invest in developing the innovation, μ_{su}^* , is characterized by*

$$\mu_{su}^* = \frac{(1 + \alpha\delta) pv_0}{(1 + 2\delta) C} - \frac{(1 + \delta)}{(1 + 2\delta)} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right). \quad (31)$$

Firms $i \in [0, \mu_{su}^]$ will invest in developing the innovation by receiving a subsidy*

$$S^* = \frac{(1 - \alpha(1 + \delta))pv_0 - (1 + \delta) \left[\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right] C}{(1 + 2\delta)(1 - \lambda)(1 - \gamma)}. \quad (32)$$

Firms $i \in (\mu_{su}^, 1]$ will also receive the subsidy S^* given in (32), but they will not invest in*

¹⁶Note that the net subsidy received by each firm in industry P, i.e., by every firm i in the interval $[0, 1]$, is $(1 - \gamma)S$. However, the part of the subsidy that does not help an innovating firm cover its shortfall between its investment costs and its expected cash flows is equal to $\lambda(1 - \gamma)S$, and this part is excluded for all firms in $[0, 1]$ from the principal's objective function since, as we explain below, it is purely an information rent paid by the principal to firms in industry P.

developing the innovation and will return a fraction $(1 - \lambda)(1 - \gamma)$ of the subsidy S^* to the principal.

(ii) The total social surplus is given by:

$$U_{su} = \frac{(1 + \alpha\delta)^2}{(1 + 2\delta)} \frac{p^2 v_0^2}{2C} - \Delta, \quad (33)$$

where

$$\Delta = \frac{(1 + \delta)}{(1 + 2\delta)} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) \left((1 - \alpha(1 + \delta))pv_0 - \frac{(1 + \delta)C}{2} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) \right).$$

The principal prefers to subsidize firms in industry P rather than not intervene, i.e., $U_{su} > U_0$, if and only if the following condition holds:

$$(1 - \lambda)(1 - \gamma) > \frac{1}{1 + \frac{(1 - \alpha(1 + \delta))pv_0}{(1 + \delta)C}}. \quad (34)$$

(iii) The equilibrium amount of innovation development μ_{su}^* is decreasing in the subsidy-skimming parameter, γ , decreasing in the retention fraction λ , and decreasing in the shadow cost of funds due to distortionary taxation, δ .

The above proposition shows that, in the case of unobservable costs, a subsidy scheme may still allow the principal to motivate a larger number of firms to invest in developing the innovation compared to the case of no intervention, i.e., $\mu_{su}^* > \mu_0$, provided that the condition given in (34) holds. This is because the principal also includes the expected payoff $(1 - \alpha)\mu pv_0$ captured by the user industry U into his objective function, so that it chooses to incentivize firms in industry P to develop the innovation. However, as we explain in detail below, subsidizing firms with unobservable costs is significantly more expensive for the principal who must not only take into account the cost of skimming (γ) and the social cost of public funds (δ), but also the cost of subsidizing firms (λ) in the interval $(\mu_{su}^*, 1]$ that have prohibitively high innovation development costs and therefore choose not to invest in developing the innovation. Further, unlike in the case of observable costs, the principal cannot condition his subsidy scheme on the cost index i of individual firms, forcing him to leave significant information rents to firms in industry P to motivate them to innovate to a greater extent.

Since a firm's innovation development cost iC is increasing in i , the incentive compatibility

constraints given in (26) and (27) imply that the incentive compatibility constraint for the marginal firm with $i = \mu_{su}^*$ given in (26) must hold as an equality in equilibrium:¹⁷

$$\alpha p v_0 - \mu_{su}^* C + (1 - \gamma) S^* = \lambda (1 - \gamma) S^*, \quad (35)$$

so that the equilibrium subsidy S^* is given by:

$$S^* = \frac{\mu_{su}^* C - \alpha p v_0}{(1 - \lambda)(1 - \gamma)}, \quad (36)$$

which is expressed in a closed-form solution in (32).

For all firms in the range $i \in (\mu_0, \mu_{su}^*]$, we know that their development cost exceeds their expected gross payoff from the innovation, i.e., $iC > \alpha p v_0$. The principal finds it optimal to subsidize these firms to cover the shortfall between their investment costs and expected cash flows, since the marginal social benefit of motivating these firms to invest in developing the innovation exceeds the marginal social cost of subsidizing their R&D investment. However, in the case of unobservable costs, the constant subsidy amount S^* given in (32) is so large that the participation constraints of firms in the interval $(\mu_0, \mu_{su}^*]$ and the firms in the interval $[0, \mu_0]$ are not binding, since the following condition holds in equilibrium:

$$\alpha p v_0 - iC + (1 - \gamma) S^* \geq \lambda (1 - \gamma) S^* > 0 \quad \forall i \in [0, \mu_{su}^*]. \quad (37)$$

One should recall that in the case of observable costs in which the principal can use targeted subsidies, the participation constraint in (10) is binding in equilibrium for all firms in the interval $(\mu_0, \mu_{so}^*]$ and firms in the interval $(\mu_{so}^*, 1]$ do not receive any subsidy at all. Further, in the case of unobservable costs, the IR constraints of firms in the interval $(\mu_{su}^*, 1]$ are also not binding as their

¹⁷Note that the reason for this is that the function in the left hand side of both constraints, $\alpha p v_0 - iC + (1 - \gamma) S$, is decreasing and continuous in i . The IC constraints in (26) and (27) also imply that the IR constraints of all firms in industry P will be satisfied as strict inequalities if the subsidy amount S is positive, since the expected payoff of each firm $i \in [0, 1]$ is at least $\lambda(1 - \gamma)S$.

expected payoff from the subsidy scheme is strictly positive as well:

$$\lambda(1 - \gamma)S^* > 0 \quad \forall i \in (\mu_{su}^*, 1]. \quad (38)$$

This explains why the subsidy mechanism is much more expensive for the principal when firms' innovation development costs are unobservable compared to the targeted subsidy mechanism when these costs are observable. Clearly, the principal's cost of subsidizing firms is increasing in the retention fraction λ , since the information rent that the principal has to pay to firms in industry P increases with the fraction of the subsidy that non-innovating firms can retain without investing in innovation development. Therefore, *ceteris paribus*, the equilibrium fraction of firms that invest in developing the innovation, μ_{su}^* , is decreasing in λ . Note also that the equilibrium subsidy S^* in (32) is positive, i.e., the principal prefers to subsidize firms rather than not intervene when costs are unobservable, if and only if the condition given in (34) holds. The LHS of this inequality is decreasing in both λ and γ . This means that, when innovation development costs are unobservable, if the retention fraction λ and the skimming parameter γ are too large, it may be too costly for the principal to intervene through a subsidy mechanism.

5.2 The Equilibrium with a Prize when Development Costs are Unobservable

In terms of its design, the prize mechanism does not depend on whether the innovation development cost structure of individual firms is observable or not. Thus, the setup of the principal's problem when he uses a prize mechanism when development costs are unobservable is identical to that in the case of observable costs. Therefore, the objective function of the principal in this case is given

by:

$$\max_{\mu, Z} (1 - \alpha)\mu pv_0 + \int_0^\mu (\alpha pv_0 - iC + pZ) di - \int_0^\mu (1 + \delta)pZ di \quad (39)$$

$$\text{s.t. } \alpha pv_0 - iC + pZ \geq 0 \quad \forall i \in [0, \mu]. \quad (40)$$

$$\alpha pv_0 - iC + pZ < 0 \quad \forall i \in (\mu, 1]. \quad (41)$$

The next proposition shows that, when the principal uses a prize mechanism, the equilibrium outcomes in the case of unobservable costs are identical to those of the prize mechanism in the case of observable costs.

Proposition 7 (*Equilibrium with a Prize when Costs are Unobservable*) *Suppose the principal offers a prize Z to incentivize firms in industry P to invest in innovation development:*

(i) *The fraction of firms that invest in developing the innovation, μ_z^* , is characterized by*

$$\mu_z^* = \frac{(1 + \alpha\delta)pv_0}{(1 + 2\delta)C}. \quad (42)$$

Firms $i \in [0, \mu_z^]$ will invest in developing the innovation. They will be candidates to win the prize Z^* , which is given by:*

$$Z^* = \frac{(1 - \alpha(1 + \delta))}{(1 + 2\delta)}v_0, \quad (43)$$

while firms $i \in (\mu_z^, 1]$ will not invest in developing the innovation and receive a zero payoff.*

(ii) *The total social surplus is given by:*

$$U_{zu} = \frac{(1 + \alpha\delta)^2 p^2 v_0^2}{(1 + 2\delta)2C}. \quad (44)$$

The principal prefers to award a prize rather than not intervene, i.e., $\mu_z^ > \mu_0$, if and only if $\alpha(1 + \delta) < 1$.*

(iii) *The equilibrium amount of innovation development μ_z^* is decreasing in the deadweight cost associated with raising taxes, δ , and increasing in the fraction of value, α , that firms in industry P capture from successful innovation development. The optimal amount of the prize offered Z^* is decreasing in δ and decreasing in α .*

When the principal decides to use a prize mechanism to motivate innovation by firms in industry P , the prize is offered *ex post* only to firms that invested in developing the innovation and collectively

succeeded in developing the innovation. The main advantage of the prize mechanism is that, when the principal cannot observe the cost structure of individual firms, this allows the principal to avoid rewarding firms which did not invest in developing the innovation, since the principal can distinguish *ex post* between firms that invested in developing the innovation and those that did not. Further, similar to the case of observable costs, when awarding a prize, the principal does not suffer from skimming costs (γ) that are incurred when subsidizing firms, since the prize is awarded *ex post* only to firms that collaborated in successfully developing the innovation.

5.3 The Comparison of Subsidies versus Prizes in the Case of Unobservable Development Costs

Next, we analyze the principal's optimal choice between subsidies versus prizes in the case of unobservable innovation development costs. The principal's objective is to incentivize firms' investment in developing the innovation by choosing the most efficient intervention mechanism available.

Proposition 8 (*The Choice between Subsidies and Prizes in the Case of Unobservable Costs*) *Let $\alpha(1+\delta) < 1$ and the condition given in (34) hold. When firms' innovation development costs are unobservable:*

- (i) *The principal will prefer to offer a prize to firms in industry P rather than to subsidize them to motivate their innovation development, i.e., $U_{zu} > U_{su}$.*
- (ii) *The fraction of firms that invest in developing the innovation when the principal uses a prize offering is greater than that when he uses a subsidy scheme, i.e., $\mu_z^* > \mu_{su}^*$.*

This proposition shows that, as long as $\gamma > 0$ and $\lambda > 0$, prizes dominate subsidies as an intervention mechanism for the principal in the case of unobservable costs. The intuition underlying this result is as follows. While a prize can be awarded only to firms that invest in developing a successful innovation *ex post*, subsidies have to be distributed to all firms in industry P regardless of whether they have invested in developing the innovation. This implies that, when firms' innovation development costs are unobservable, the government's cost of using subsidies to motivate firms to innovate is much larger than its cost of using a prize offering to achieve a greater extent of

innovation development across firms in industry P. Therefore, in equilibrium, the fraction of firms that invest in developing the innovation in the case of a prize is significantly larger than that in the case of a subsidy scheme, i.e., $\mu_z^* > \mu_{su}^*$. This means that user firms in industry U can expect to benefit from the positive externality generated by a successful innovation to a greater extent when the principal uses a prize mechanism rather than a subsidy mechanism in the case of unobservable costs. Therefore, from the point of view of the principal who wants to maximize the total surplus in the economy, prizes clearly dominate subsidies when firms' cost structures are unobservable. This result stands in stark contrast with our earlier result in the comparison of subsidies versus prizes in the case of observable development costs, in which a targeted provision of subsidies conditional on firms' development costs dominates the prize mechanism in many instances.

This proposition is consistent with practice, where, in the case of path-breaking innovations in which governments or non-profit foundations are not only unaware of the development cost structure, but do not even know the set of firms or individuals who may be able to successfully develop the innovation, usually a prize, awarded only to successful innovators, is used to motivate the development of these innovations. Clearly, in such a situation, a subsidy scheme would be prohibitively expensive, since, in this case, the principal would have to subsidize all firms and individuals who are even remotely capable of developing the innovation (and who apply for such subsidies).

6 The Role of Venture Capital in Fostering Innovation

In this section, we extend our basic model to introduce another player into the game: the venture capitalist (VC). Our objective is to analyze and understand the role of venture capital firms in motivating entrepreneurial firms to invest in innovation development, and how this interacts with the government's role that we analyzed in our basic model.¹⁸

¹⁸In our basic model, firms in industry P can be thought as both small, high-growth oriented, entrepreneurial firms and large, mature firms. In this extended model, it is more natural to think of firms in industry P as being entrepreneurial, high-growth, private firms with considerable potential for producing innovations, since we specifically

We assume that the VC has existing equity holdings in some portfolio companies from the user industry U so that she has an aggregate claim against a fraction β of the total cash flows produced by firms in that industry. For analytical simplicity, the value of a firm in industry P depends solely on the value of its innovation project.¹⁹ Further, we assume that if the VC decides to invest in a company i in the producer industry P, she finances a fraction $(1 - \rho)$ of the required innovation development cost iC in exchange for an equity stake in the company, while the entrepreneur invests the remaining fraction ρ . For analytical simplicity, we assume that the entrepreneurs of firms in industry P are solely invested in their own companies, and that $\rho < \frac{\alpha p v_0}{C}$.

6.1 The Case of No Government Intervention

In this section, we analyze the case in which there is no government intervention, and the VC is the only agent who may play a role in incentivizing innovation. From our analysis of the competitive private sector equilibrium in Section 3.2, we know that for firms with a cost index in the interval $[0, \mu_0]$, the NPV of investment in innovation development is positive, i.e., $\alpha p v_0 - iC > 0$ if $i \in [0, \mu_0]$. Therefore, firms with $i \in [0, \mu_0]$ will invest in developing the innovation. We assume that the entire NPV of the innovation development is captured by the entrepreneurs of these firms. It follows that the equity stakes of the entrepreneur and the VC respectively, in a company with cost index $i \in [0, \mu_0]$ are given by:

$$s_e^i = \frac{\rho i C + (\alpha p v_0 - i C)}{\alpha p v_0}, \quad s_v^i = \frac{(1 - \rho) i C}{\alpha p v_0}, \quad \forall i \in [0, \mu_0], \quad (45)$$

respectively.

Note that the VC benefits from the fact that an increasing number of firms in the producer industry P invest in developing the innovation, since she becomes more likely to capture a fraction

analyze the role of venture capitalists in fostering innovation.

¹⁹Even if we relax this assumption, and assume that assets in place (other projects) with known values, all our results will go through qualitatively unchanged.

β of the surplus that firms in the user industry U enjoy from a successful innovation developed by firms in industry P. Due to this externality benefit of innovation, the VC can also decide to finance some additional number of companies in industry P to induce them to develop the innovation. Such firms would have otherwise not invested in developing the innovation without the involvement of the VC, since the NPV of innovation development is negative for them, i.e., $\alpha p v_0 - iC < 0$ for all firms with $i \in (\mu_0, 1]$. In this case, to increase his expected payoff from his equity holdings in the user industry U, the VC may be willing to bear the negative NPV of the innovation development of the firms in the interval $(\mu_0, \mu_v]$, and give the entrepreneurs of these firms disproportionately higher equity stakes to ensure that they contribute a fraction ρ of the investment required for developing the innovation. This means that the equity stakes of the entrepreneur and the VC in a firm with cost index $i \in (\mu_0, \mu_v]$ have to satisfy the following conditions:

$$s_e^i \geq \frac{\rho i C}{\alpha p v_0}, \quad s_v^i \leq \frac{(1 - \rho) i C + (\alpha p v_0 - i C)}{\alpha p v_0}, \quad \forall i \in (\mu_0, \mu_v], \quad (46)$$

respectively. Thus, the VC's expected payoff from investing an amount $(1 - \rho)iC$ in each of these firms in the producer industry P is negative, and equal to

$$-(1 - \rho)iC + s_v^i (\alpha p v_0) \leq \alpha p v_0 - iC < 0, \quad \forall i \in (\mu_0, \mu_v]. \quad (47)$$

In this setting, in which there is no intervention by the government, the VC will solve the following problem to maximize her expected payoff:

$$\max_{\mu_v, s_v^i} \int_{\mu_0}^{\mu_v} (-(1 - \rho)iC + s_v^i (\alpha p v_0)) di + \beta(1 - \alpha)\mu_v p v_0 \quad (48)$$

$$\text{s.t. } (1 - s_v^i) \alpha p v_0 - \rho i C \geq 0, \quad \forall i \in (\mu_0, \mu_v]. \quad (49)$$

The first term in this objective function is the VC's expected payoff from investing in firms with

a cost index i greater than μ_0 and motivating them to invest in developing the innovation. The second term is the VC's expected payoff from a successful innovation that she captures by way of his equity holdings in firms in the user industry U. The participation constraint of the entrepreneurs of firms with $i \in (\mu_0, \mu_v]$ is given in (49), and it follows from the conditions given in (46). The VC's objective is to determine the optimal cutoff point μ_v , below which she will invest in firms in industry P, and to determine her optimal equity stakes in the portfolio companies that she plans to invest in.

Proposition 9 (*Private Sector Equilibrium with Venture Capital*) *Suppose that the principal does not intervene in industry P, but the VC holds existing equity stakes in portfolio companies from the user industry U while investing new funds in firms in the producer industry P.*

- (i) *The fraction of firms that invest in developing the innovation in industry P, μ_v^* , is characterized by*

$$\mu_v^* = (\alpha + \beta(1 - \alpha)) \frac{pv_0}{C}. \quad (50)$$

Firms $i \in [0, \mu_v^]$ will invest in developing the innovation, while firms $i \in (\mu_v^*, 1]$ will not invest in developing the innovation.*

- (ii) *The VC's equity stake in firms in industry P is given by:*

$$s_v^{i,*} = \begin{cases} \frac{(1-\rho)iC}{\alpha pv_0}, & \text{if } i \in [0, \mu_0], \\ \frac{(1-\rho)iC + (\alpha pv_0 - iC)}{\alpha pv_0}, & \text{if } i \in (\mu_0, \mu_v^*], \\ 0, & \text{if } i \in (\mu_v^*, 1]. \end{cases} \quad (51)$$

- (iii) *The fraction of firms that invest in developing the innovation in industry P in the private sector equilibrium with venture capital is greater than that in the private sector equilibrium without venture capital, i.e., $\mu_v^* > \mu_0$.*

This proposition shows that the presence of venture capital backing in a producer industry can help motivate entrepreneurial firms in that industry to innovate to a greater extent compared to the private sector equilibrium in our basic model in which venture capital plays no role, i.e., $\mu_v^* > \mu_0$. Because the VC holds claim to a fraction β of cash flows produced in the user industry U, she is able to internalize the positive externality of a successful innovation produced by his portfolio companies in the producer industry P. This creates an incentive for the VC firm to provide financial backing to some entrepreneurial firms in industry P to motivate them to invest in developing the innovation,

which would have otherwise not invested in innovation development (without the backing of the VC). One should also note that the equilibrium fraction of firms that invest in developing the innovation with the financial backing of the VC in industry P, μ_v^* given in (50), increases with the equity ownership of the VC in the user industry U, i.e., β . In summary, since the VC diversifies her investments in portfolio companies across the producer and the user industries, this allows her to benefit from positive externalities and spill-over effects of innovations that are produced by her portfolio companies from the producer to the user industry.

We know that entrepreneurs of firms with $i \leq \mu_0$ will choose to invest in developing the innovation on their own, since it is a positive-NPV investment for their firms. Therefore, the VC invests in the equity of these firms at competitive rates while the entrepreneurs capture their positive NPV. However, investments in innovation development in firms with a cost index greater than μ_0 have a negative NPV. Thus, the entrepreneurs of these firms would not have invested in innovation development without the VC providing cheap funds to them. Nevertheless, as we explained above, the VC finds it profitable to motivate entrepreneurial firms in the interval $(\mu_0, \mu_v^*]$ to innovate as well, and the least-cost way for the VC to accomplish that is to ensure that the participation constraint given in (49) is binding for these firms. Therefore, the entrepreneurs of these firms retain a sufficiently large equity stake so that they just break even by agreeing to invest in developing the innovation. Thus, the entrepreneurs' and the VC's equity stakes in such firms are given by:

$$s_e^i = \frac{\rho i C}{\alpha p v_0}, \quad s_v^{i,*} = \frac{(1 - \rho) i C + (\alpha p v_0 - i C)}{\alpha p v_0}, \quad (52)$$

respectively, for all firms with $i \in (\mu_0, \mu_v^*]$. The VC thus provides funds to these firms at a preferentially low cost of equity; in other words, the VC is willing to incur the negative NPV of innovation development for such firms, which is equal to $\alpha p v_0 - i C < 0$ for all $i \in (\mu_0, \mu_v^*]$.

Given the equilibrium equity stakes of the VC in (51), it is straightforward to show that the

equilibrium amount of innovation μ_v^* in (50) is also the solution to the following problem:

$$\max_{\mu} \int_0^{\mu} (\alpha p v_0 - i C) di + \beta(1 - \alpha)\mu p v_0. \quad (53)$$

Thus, if the fraction of firms that invest in developing the innovation is equal to μ_v^* , the expected payoffs to both the entrepreneurs in industry P and the VC are jointly maximized.

6.2 The Interaction of Venture Capital and Government when Development Costs are Observable

In this section, we analyze a setting in which the VC and the government (the principal) simultaneously engage in motivating entrepreneurial firms in industry P to innovate. In our above analysis of subsidies in the case of observable costs in Section 4, we showed that the government can use a targeted subsidy scheme to significantly increase the fraction of firms that innovate compared to the private sector equilibrium. When the government notices that VC firms can help firms in industry P to partially internalize the positive externality of an innovation as we showed in Proposition 9, it will realize that it can further fine-tune its subsidy scheme to lower its overall cost of subsidizing firms while achieving the second-best social optimum.

In this section, we assume that the only inefficiency that the government faces when subsidizing firms in industry P is the social cost of funds ($\delta > 0$) associated with the deadweight losses of taxation. Then, from (12), it follows that the socially optimal fraction of firms that invest in developing the innovation without the involvement of the VC is equal to

$$\mu_{so}^* = \left(\alpha + \frac{1}{(1 + \delta)}(1 - \alpha) \right) \frac{p v_0}{C}. \quad (54)$$

If the fraction of the user industry cash flows appropriated by the VC, β , is less than $\frac{1}{1 + \delta}$, then it follows that $\mu_v^* < \mu_{so}^*$. In this case, the next proposition shows that the government may be able to

intervene to motivate additional firms in industry P to invest in developing the innovation, so as to bring the fraction of innovating firms to the social optimum.

Proposition 10 (*Equilibrium with Venture Capital and Government Intervention when Costs are Observable*) Let $\beta < \frac{1}{(1+\delta)}$. If the VC holds existing equity stakes in portfolio companies from the user industry U while investing new funds in firms in the producer industry P, and the principal intervenes in industry P using subsidies, the following Nash equilibrium can be supported:

- (i) The entrepreneurs of firms $i \in [0, \mu_0]$ will invest in developing the innovation without receiving a subsidy from the government. The VC provides a fraction $(1 - \rho)$ of the required investment iC at competitive terms in exchange for a fraction of firm i 's equity, which is equal to

$$s_v^{i,*} = \frac{(1 - \rho)iC}{\alpha p v_0}. \quad (55)$$

- (ii) The entrepreneurs of firms $i \in (\mu_0, \mu_v^*]$ will invest in developing the innovation without receiving a subsidy from the government. The VC provides a fraction $(1 - \rho)$ of the required investment iC on concessional terms in exchange for a fraction of firm i 's equity, which is equal to

$$s_v^{i,*} = \frac{(1 - \rho)iC + (\alpha p v_0 - iC)}{\alpha p v_0}. \quad (56)$$

- (iii) The entrepreneurs of firms $i \in (\mu_v^*, \mu_{vg}^*]$ will invest in developing the innovation after receiving a subsidy S_i^* from the government, which is equal to

$$S_i^* = iC - \alpha p v_0. \quad (57)$$

The VC will not invest in these firms.

- (iv) The total fraction of firms that invest in developing the innovation in industry P, μ_{vg}^* , in equilibrium is characterized by

$$\mu_{vg}^* = \left(\alpha + \frac{1}{(1 + \delta)}(1 - \alpha) \right) \frac{p v_0}{C}, \quad (58)$$

where μ_{vg}^* is also equal to μ_{so}^* given in (54).

The above proposition shows that, in order to motivate entrepreneurial firms in the interval $(\mu_0, \mu_{vg}^*]$ to innovate, the VC and the government can provide financing to firms in different cost segments of this sub-market, i.e., in the intervals $(\mu_0, \mu_v^*]$ and $(\mu_v^*, \mu_{vg}^*]$, respectively. The intuition underlying this result is as follows. The government knows that without the government providing subsidies, the VC is willing to provide financial backing to firms in the interval $(\mu_0, \mu_v^*]$ as shown in Proposition 9. Thus, instead of providing subsidies to all firms in the interval $(\mu_0, \mu_{vg}^*]$, the

government can choose the strategy of precommitting to subsidize only firms with a cost index greater than μ_v^* . Given the VC's equilibrium strategy given in parts (i) and (ii) of the above proposition, this strategy is optimal for the government, since this helps the government to minimize its total cost of subsidizing firms while achieving the same socially second-best optimal amount of innovation, i.e., $\mu_{vg}^* = \mu_{so}^*$.

In the Nash equilibrium given in Proposition 10, the VC solves the following optimization problem given the equilibrium strategies of other players:

$$\max_{\mu \geq \mu_0, s_v^i} \int_{\mu_0}^{\mu} \left(-(1-\rho)iC + s_v^i(\alpha p v_0) \right) di + \beta(1-\alpha) \left(\mu + (\mu_{vg}^* - \mu_v^*) \right) p v_0 \quad (59)$$

$$\text{s.t. } (1 - s_v^i) \alpha p v_0 - \rho i C = 0, \quad \forall i \in (\mu_0, \mu]. \quad (60)$$

The optimal solution to the above problem of the VC is the same as that for the problem given in (48), i.e., $\mu^* = \mu_v^*$. Given the government's equilibrium strategy of subsidizing only firms in the interval $(\mu_v^*, \mu_{vg}^*]$, it still remains optimal for the VC to finance up to a fraction μ_v^* of firms in industry P, since his trade-off between the marginal benefit of investing in another firm with a cost index above μ_0 and the marginal cost of doing so remains the same.

In the above equilibrium, the entrepreneurs of firms with $i \in [0, \mu_0]$ invest in developing the innovation without receiving a subsidy from the government, and the VC invests in these firms at competitive terms so that the entire positive NPV is captured by the entrepreneurs, who provide a fraction ρ of the required investment iC . In firms with $i \in (\mu_0, \mu_v^*]$, the entrepreneurs participate in developing the innovation because the VC provides external funds to these firms at low-cost preferential rates so that the entrepreneurs just break even by investing a fraction ρ of the required investment iC . Finally, entrepreneurs of firms with $i \in (\mu_v^*, \mu_{vg}^*]$ are willing to invest the required investment iC for developing the innovation, since the government allows them to break even by providing a subsidy S_i^* in (57) that covers the shortfall between the investment cost iC and the expected cash flow $\alpha p v_0$.

6.3 The Interaction of Venture Capital and Government when Development Costs are Unobservable

In contrast to the previous section in which both the VC and the government could observe the innovation development costs of all firms in industry P, we now assume that these costs are observable only to the VC, but not to the government. For tractability, we assume that there are three dates and the VC invests in individual firms in industry P at date 0. Because of its business expertise and industry focus, we assume that the VC has already incurred some sunk costs to uncover information about the cost structure of individual firms in industry P (so that she can observe firms' innovation development cost structure), whereas the government has no information at all about the cost structure of these firms at date 0. Further, we assume that the government can observe all the firms that the VC has invested in at date 0 (and their costs), and will intervene only subsequently, at date 1, to motivate additional firms to invest in developing the innovation. Uncertainty about the development of the innovation is resolved at date 2, and if the innovation is developed successfully by date 2, its value v_0 is also realized at date 2.

The government realizes that without its intervention at date 1, the VC would invest in all firms in the interval $[0, \mu_v^*]$ at date 0, where μ_v^* is given by (50). For simplicity, we assume that the government can precommit not to intervene at date 1 unless it has observed that the VC has invested in all firms in the interval $[0, \mu_v^*]$ at date 0.²⁰ Given the above assumptions, the government's problem at date 1 is to decide on the additional fraction of firms it wants to motivate to develop the innovation in the interval $(\mu_v^*, 1]$.

One should note that, even after observing all the firms that the VC has invested in at date 0, the government is still unable to observe the cost structure of firms with $i \in (\mu_v^*, 1]$. In this setting,

²⁰With this assumption, we rule out cases in which the VC has an incentive to free-ride on the government's efforts to motivate innovation by not investing in firms with a cost index greater than μ_0 . In practice, many governments are financially and politically constrained to keep their budget deficits within reasonable limits and prevent run-ups in their spending financed by taxes. This lends credence to the government's precommitment not to intervene at date 1 unless the VC invested in all firms in the interval $[0, \mu_v^*]$ at date 0. Further, in some countries, an extensive interventionist industrial policy by the government may also be politically controversial, thus limiting the resources that the government can commit to such interventions.

the government can achieve its objective of motivating additional firms to innovate by utilizing one of two different methods. First, it can provide untargeted subsidies to all firms in the interval $(\mu_v^*, 1]$ directly, using a scheme similar to that we analyzed in Section 5.1. We call this as the benchmark method. Second, the government can optimally channel some of its funds to the VC, who, in turn, can provide targeted subsidies to some firms in the interval $(\mu_v^*, 1]$ in exchange for fees from the government. The government may use some remaining funds to subsidize additional firms directly as well, using the same approach as in the benchmark method, but for a smaller subset of firms.

6.3.1 The Case where the Government Uses Direct Subsidies Alone

If the government adopts the benchmark method and directly offers the same amount of subsidy S to all firms in the interval $(\mu_v^*, 1]$ at date 1, it solves the following problem:

$$\begin{aligned} \max_{\mu, S} \quad & (1 - \alpha)\mu p v_0 + \int_0^{\mu_v^*} (\alpha p v_0 - iC) di + \int_{\mu_v^*}^{\mu} (\alpha p v_0 - iC + (1 - \lambda)S) di \\ & - (1 + \delta) \left(\int_{\mu_v^*}^{\mu} S di + \int_{\mu}^1 \lambda S di \right) \\ \text{s.t.} \quad & \alpha p v_0 - iC + S \geq \lambda S \quad \forall i \in (\mu_v^*, \mu]. \end{aligned} \tag{61}$$

In the above objective function, the first term is the expected externality of the innovation appropriated by the firms in the user industry U. The second term is the expected payoff of all the firms that the VC invests in due to innovation development. The third term is the expected payoff of all the government-subsidized firms in the interval $[\mu_v^*, \mu]$ that invest in developing the innovation, net of the information rent λS they receive. Finally, the fourth term is the government's cost of directly subsidizing all firms in the interval $[\mu_v^*, 1]$. The IC constraints of innovating firms in the interval $[\mu_v^*, \mu]$ is given by (61). The next proposition shows how the government and the VC can both motivate firms in industry P to invest in the development of the innovation when the costs are observable to the VC, but unobservable to the government.

Proposition 11 (The Benchmark Equilibrium with Venture Capital and Government Intervention) *Let the VC hold existing equity stakes in portfolio companies from the user industry U with $\beta < \frac{1}{(1+\delta)}$, and let $\lambda < \bar{\lambda}$, where $\bar{\lambda}$ is given in (A.79). If the VC invests new funds in firms in the producer industry P at date 0, and the government intervenes in industry P using untargeted subsidies at date 1, the following equilibrium can be supported:*

- (i) *The entrepreneurs of firms $i \in [0, \mu_0]$ will invest in developing the innovation at date 0 without receiving a subsidy from the government. The VC provides a fraction $(1 - \rho)$ of the required investment iC at competitive terms in exchange for a fraction of firm i 's equity, which is equal to*

$$s_{vu}^{i,*} = \frac{(1 - \rho)iC}{\alpha p v_0}. \quad (62)$$

- (ii) *The entrepreneurs of firms $i \in (\mu_0, \mu_v^*]$ will invest in developing the innovation at date 0 without receiving a subsidy from the government. The VC provides a fraction $(1 - \rho)$ of the required investment iC on concessional terms in exchange for a fraction of firm i 's equity, which is equal to*

$$s_{vu}^{i,*} = \frac{(1 - \rho)iC + (\alpha p v_0 - iC)}{\alpha p v_0}. \quad (63)$$

- (iii) *The entrepreneurs of firms $i \in (\mu_v^*, \mu_{gu}^*]$ will invest in developing the innovation at date 1 after receiving a subsidy $S^* = \frac{\mu_{gu}^* C - \alpha p v_0}{1 - \lambda}$ from the government. The VC will not invest in these firms.*

- (iv) *The total fraction of firms that invest in developing the innovation in industry P , μ_{gu}^* , is characterized by*

$$\mu_{gu}^* = \frac{(1 + \alpha\delta) p v_0}{(1 + 2\delta) C} + \frac{(\lambda + \delta)}{(1 - \lambda)(1 + 2\delta)} \mu_v^* - \frac{\lambda(1 + \delta)}{(1 - \lambda)(1 + 2\delta)}. \quad (64)$$

This proposition characterizes the optimal fraction of firms that invest in developing the innovation, μ_{gu}^* , as a solution to the problem given in (61). Substituting μ_v^* from (50) in (64), we obtain:

$$\mu_{gu}^* = \frac{(1 + \delta(\alpha(2 - \beta) + \beta) - \lambda((1 - \alpha)(1 - \beta) + \delta\alpha)) p v_0}{(1 - \lambda)(1 + 2\delta) C} - \frac{\lambda(1 + \delta)}{(1 - \lambda)(1 + 2\delta)}. \quad (65)$$

It is straightforward to show that the IC constraint in (61) will be binding only for the marginal firm with $i = \mu_{gu}^*$, so that the uniform subsidy level S^* is determined by the firm with the highest cost $\mu_{gu}^* C$ among innovating firms:

$$S^* = \frac{(((1 - \alpha)(1 - \beta\delta) - \lambda(1 - \alpha(2 + \delta) - \beta(1 - \alpha)))) p v_0 - \lambda(1 + \delta)C}{(1 - \lambda)^2(1 + 2\delta)}. \quad (66)$$

This means that since the development costs of the firms in the interval $(\mu_v^*, 1]$ are still unobservable to the government at date 1, it has to offer large information rents to the subsidized firms in the interval $(\mu_v^*, \mu_{gu}^*]$ to motivate them to invest in developing the innovation. Further, the efficiency with which the government subsidizes innovation development decreases with λ , which is the fraction of the subsidy that firms in the interval $(\mu_{gu}^*, 1]$ capture even though they have prohibitively high development costs and therefore do not invest in developing the innovation (while they still apply for and receive the subsidy from the government). Therefore, the optimal level of innovation μ_{gu}^* in the benchmark case given in (64) is decreasing in λ , and it is clearly less than the socially optimal level of innovation μ_{vg}^* (given in (58)) in the previous section in which the government could observe the cost structure of all firms in industry P.

The above proposition shows that, when the development costs of firms are unobservable to the government, the amount of innovation that the VC and the government can motivate together by using the benchmark method we describe above is greater than the level of innovation that the government can motivate on its own by providing the same level of subsidy to all firms in industry P (as in Section 5.1): i.e., it is straightforward to verify that $\mu_{gu}^* > \mu_{su}^*$ when $\gamma = 0$. This is the case mainly because in this benchmark equilibrium, firms in the interval $[0, \mu_v^*]$ do not receive untargeted government subsidies. Instead, they are more efficiently financed by their entrepreneurs with the help of VC financing. Moreover, despite the inefficiencies associated with untargeted government subsidies, the above proposition also shows that, if the parameter λ is sufficiently small so that $\lambda < \bar{\lambda}$, the government is able to motivate an additional mass of firms to innovate through direct (and constant) subsidies to some firms, over and above the fraction of firms that the VC invests in: i.e., $\mu_{gu}^* > \mu_v^*$ when $\lambda < \bar{\lambda}$.

Corollary 1 *Let $\beta < \frac{1}{(1+\delta)}$ and $\lambda = 0$. Then, the total fraction of firms that invest in developing the innovation in industry P, μ_{gu}^* , in the above benchmark equilibrium in Proposition 11 is equal to*

$$\mu_{gu}^* = \frac{(1 + \alpha\delta)pv_0 + \delta C\mu_v^*}{(1 + 2\delta)C} = \frac{(1 + \delta(\alpha(2 - \beta) + \beta))}{(1 + 2\delta)} \frac{pv_0}{C}, \quad (67)$$

where $\mu_{gu}^* > \mu_v^* = \frac{(\alpha+\beta(1-\alpha))pv_0}{C}$ and $\mu_{gu}^* < \mu_{vg}^* = \frac{(1+\alpha\delta)pv_0}{(1+\delta)C}$.

The above corollary shows that, even if we assume that non-innovating firms in the interval $(\mu_{gu}^*, 1]$ return the entire subsidy S to the government at date 2 (i.e., $\lambda = 0$), the amount of total innovation that the VC and the government can motivate together in the case of unobservable costs, μ_{gu}^* , is still less than the socially optimal level of innovation, μ_{vg}^* , that they can generate in the case of observable costs. This is due to the government's inability to provide targeted subsidies to firms in the interval $(\mu_v^*, \mu_{gu}^*]$ in the case of unobservable costs, which increases the government's cost of motivating firms to innovate.

6.3.2 The Case where the Government Channels some Subsidies through Venture Capital Funds

In this section, we analyze the case where the government utilizes the services of the VC to indirectly subsidize some firms in the interval $(\mu_v^*, 1]$ at date 1 in order to achieve a more efficient outcome compared to that obtained in the benchmark equilibrium we discussed in the previous section. In this alternative approach, the government channels an optimally determined fraction of its subsidy funds to the VC and asks it to provide targeted subsidies S_i to firms in the interval $(\mu_v^*, \mu_1]$ as a function of the firm's innovation development cost iC . This cost is observable to the VC, but unobservable to the government at date 1.²¹ We assume that, in exchange for providing this service, the VC charges the government a fee f_i for each firm $i \in (\mu_v^*, \mu_1]$ that it subsidizes on behalf of the government at date 1 according to the following schedule:

$$f_i = f + b(i - \mu_v^*) \quad \forall i \in (\mu_v^*, \mu_1], \quad (68)$$

²¹We assume that, while the government does not observe the innovation development costs of individual firms, it is able to observe the number of firms the VC invests in and the total amount of funds invested. Given this, it should be clear that the VC will not benefit from investing government funds in the wrong (high-cost) firms: recall that the VC herself has an incentive to maximize the successful development of the innovation, given her equity holdings in firms in the user industry U.

where $f > 0$ is the fixed fee cost for each firm, and $b \geq 0$ is the marginal fee charged for each additional firm the government subsidizes through the VC.²²

After observing that the VC channels the government's subsidies on its behalf to the firms in the interval $(\mu_v^*, \mu_1]$, the government then uses its remaining subsidy funds to directly subsidize all firms in the interval $(\mu_1, 1]$ as it does in the benchmark method. Thus, each firm in the interval $(\mu_1, \mu]$ receives the full uniform subsidy S and invests in developing the innovation. Firms in the interval $(\mu, 1]$ receive the same subsidy, but do not invest in developing the innovation even though each of them retains a fraction λ of the subsidy S (and returns the remaining fraction back to the government). In this setting, the government solves the following problem:

$$\begin{aligned} \max_{\mu, \mu_1, S_i, S} \quad & \int_0^{\mu_v^*} (\alpha p v_0 - iC) di + \int_{\mu_v^*}^{\mu_1} (\alpha p v_0 - iC + S_i + f + b(i - \mu_v^*)) di \\ & + \int_{\mu_1}^{\mu} (\alpha p v_0 - iC + (1 - \lambda)S) di + (1 - \alpha)\mu p v_0 \\ & - (1 + \delta) \left(\int_{\mu_v^*}^{\mu_1} (S_i + f + b(i - \mu_v^*)) di + \int_{\mu_1}^{\mu} S di + \int_{\mu}^1 \lambda S di \right) \end{aligned} \quad (69)$$

$$\text{s.t.} \quad \alpha p v_0 - iC + S_i \geq 0 \quad \forall i \in (\mu_v^*, \mu_1], \quad (70)$$

$$\alpha p v_0 - iC + S \geq \lambda S \quad \forall i \in (\mu_1, \mu], \quad (71)$$

$$\mu_v^* < \mu_1 < \mu. \quad (72)$$

In the above objective function, the first term is the expected payoff of all innovating firms financed by the VC. The second term is the sum of the expected payoff of all firms ($i \in (\mu_v^*, \mu_1]$) subsidized indirectly by the VC on behalf of the government and the VC's fee payoff. The third term is

²²For tractability, our model assumes the existence of a representative VC. Clearly, this VC benefits from the government's efforts to subsidize additional firms (with a cost index greater than μ_v^*) to invest in innovation development at date 1. Therefore, some readers might wonder as to why the VC would charge fees to the government to subsidize these firms more efficiently on its behalf. In practice, note that the VC industry consists of many different, competitive VC firms with a substantial amount of heterogeneity in their portfolio firm investments and incentives. Therefore, they would charge fees to any investor who wants to benefit from their private information about the cost structure of their potential portfolio firms. Further, as the number of portfolio firms the government subsidizes with the help of the VCs increases, it is reasonable to expect that the VC industry's marginal cost of providing this service to the government increases, i.e., $b > 0$. Our assumption of the above linear fee schedule reflects these real-world aspects of the VC industry. Note also that this assumption also covers the special case in which the VC charges only a fixed fee f for each firm subsidized using her services to the government, i.e., $b = 0$; our results hold qualitatively unchanged even for this special case.

the expected payoff of all directly government-subsidized firms in the interval $[\mu_v^*, \mu]$ that invest in developing the innovation, net of the information rent λS they receive. The fourth term is the expected externality of the innovation, and finally, the fifth term is the government's cost of subsidizing all firms in the interval $[\mu_v^*, 1]$. The participation constraints of firms subsidized by the government indirectly through the VC is given in (70), and the IC constraints of innovating firms subsidized directly by the government is given in (71).²³

The government's optimization problem in (69) to (72) can be solved in two steps. First, it is straightforward to show that at the optimal solution, the participation constraints in (70) will be binding for all firms with $i \in (\mu_v^*, \mu_1]$, and the IC constraint in (71) will be binding only for the marginal firm with $i = \mu$. Further, the above optimization problem also implies that, for a given value of μ , the optimal value of μ_1 must solve the following problem:

$$\min_{\mu_1, S_i, S} \int_{\mu_v^*}^{\mu_1} \delta (S_i + f + b(i - \mu_v^*)) di + \int_{\mu_1}^{\mu} (\lambda + \delta) S di \quad (73)$$

$$\text{s.t. } S_i = iC - \alpha p v_0 \quad \forall i \in (\mu_v^*, \mu_1], \quad (74)$$

$$S = \frac{\mu C - \alpha p v_0}{(1 - \lambda)}. \quad (75)$$

Substituting S_i from (74) and S from (75) into the objective function in (73), and solving for the first order condition of this subproblem with respect to μ_1 yields:

$$\mu_1 = \frac{b}{(C + b)} \mu_v^* + \left(\frac{1 + \frac{\lambda}{\delta}}{1 - \lambda} \right) \frac{C}{(C + b)} \mu + \left(1 - \frac{1 + \frac{\lambda}{\delta}}{1 - \lambda} \right) \frac{\alpha p v_0}{C} - \frac{f}{(C + b)}. \quad (76)$$

The solution to the above problem in (73) to (75) reflects the trade-off the government faces when deciding what fraction of innovating firms in the interval $(\mu_v^*, \mu]$ to subsidize through the VC.

²³It follows from the constraint given in (72) that in the above problem, we don't allow μ_1 to take the corner solution value of μ . This is because when $\mu_1 = \mu$, the government does not provide any direct subsidies to firms so that the last term in the objective function given in (69), $(1 + \delta) \left(\int_{\mu}^1 \lambda S di \right)$, suddenly drops out even though it is positive as long as $\lambda > 0$ and $\mu < 1$. Since this creates a discontinuity of the above objective function at $\mu_1 = \mu$, we consider the government's problem and the solution to this problem separately when $\mu_1 = \mu$.

Starting from the firm with the lowest cost index $i = \mu_v^*$ and going up the cost ladder over the interval $(\mu_v^*, \mu_1]$, the government's targeted subsidy S_i to each firm i through the VC covers an increasing innovation development cost iC while the firm i 's expected cash flow from the innovation, $\alpha p v_0$, is constant. Thus, the net benefit of subsidizing each firm indirectly through the VC decreases as i increases from μ_v^* to μ_1 . Further, in addition to the fixed fee f per firm, the government also pays a positive marginal fee b for each additional firm it subsidizes through the VC so that its total fee cost increases quadratically as i increases. On the other hand, for each firm in the interval $(\mu_1, \mu]$ that it subsidizes directly, the government does not pay any fees to the VC. However, it has to pay a uniform subsidy S that covers the cost μC of the marginal firm μ , which has the highest cost index among innovating firms. Thus, due to its inability to observe the cost structure of firms in the interval $(\mu_1, \mu]$, the government leaves a rent of $(\mu - i)C$ to each of these firms.²⁴ Essentially, the cutoff point μ_1 in (76) is determined by comparing the variable cost $(iC + f + b(i - \mu_v^*))$ to the fixed cost μC . Clearly, this cutoff point is decreasing in the fixed fee f paid to the VC for each firm in $(\mu_v^*, \mu_1]$, and it is closer to μ_v^* as the marginal fee b becomes higher. Further, for a given value of μ , the cutoff value μ_1 is increasing in the parameter λ . Thus, the higher the partial subsidization of non-innovating firms in the interval $(\mu, 1]$, the higher the information rent the government has to pay to innovating firms in the interval $(\mu_1, \mu]$ to satisfy their incentive compatibility constraints. Therefore, to reduce this inefficiency, the government chooses a higher cutoff value μ_1 for higher values of λ . The second step to the solution of the government's optimization problem in (69) to (72) involves substituting the value of μ_1 from (76) into the objective function in (69) and solving for the optimal value of μ .

Finally, if the government provides all of its subsidies through the VC (i.e., indirectly), it solves

²⁴The expected cash flow from innovation $\alpha p v_0$ is also constant for each firm $i \in (\mu_1, \mu]$.

the following problem:²⁵

$$\begin{aligned}
\max_{\mu, S_i} \quad & (1 - \alpha)\mu p v_0 + \int_0^\mu (\alpha p v_0 - iC) di + \int_{\mu_v^*}^\mu (S_i + f + b(i - \mu_v^*)) di \\
& - (1 + \delta) \int_{\mu_v^*}^\mu (S_i + f + b(i - \mu_v^*)) di \\
\text{s.t.} \quad & \alpha p v_0 - iC + S_i \geq 0 \quad \forall i \in (\mu_v^*, \mu],
\end{aligned} \tag{77}$$

which allows it to determine the optimal total fraction of firms to motivate to develop the innovation.

The next proposition characterizes the above equilibrium in which the government channels an optimally determined fraction of its subsidies to the firms in the interval $(\mu_v^*, \mu_1]$ through the VC and the remaining fraction of its subsidies directly in the form of a flat (i.e., independent of development costs) subsidy to other firms (with $i > \mu_1$) in order to motivate innovation development in the producer industry.

Proposition 12 (*The Equilibrium with the Government Channeling Subsidies Partially through the VC*) *Let the VC hold existing equity stakes in portfolio companies from the user industry U with $\beta < \frac{1}{(1+\delta)}$. Further, let $\lambda < \bar{\lambda}$ and $f_1 < f < f_2$, where the lower threshold f_1 is defined by (A.88) and the upper threshold f_2 is given in (A.85). If the VC invests new funds in firms in the producer industry P at date 0, and the government intervenes in industry P using both targeted subsidies channeled through the VC and untargeted direct subsidies directly invested in firms at date 1, the following equilibrium exists:*

- (i) *The entrepreneurs of firms $i \in [0, \mu_0]$ will invest in developing the innovation at date 0 without receiving a subsidy from the government. The VC provides a fraction $(1 - \rho)$ of the required investment iC at competitive terms in exchange for a fraction of firm i 's equity, which is equal to*

$$s_{vu}^{i,*} = \frac{(1 - \rho)iC}{\alpha p v_0}. \tag{78}$$

- (ii) *The entrepreneurs of firms $i \in (\mu_0, \mu_v^*]$ will invest in developing the innovation at date 0 without receiving a subsidy from the government. The VC provides a fraction $(1 - \rho)$ of the required investment iC on concessional terms in exchange for a fraction of firm i 's equity, which is equal to*

$$s_{vu}^{i,*} = \frac{(1 - \rho)iC + (\alpha p v_0 - iC)}{\alpha p v_0}. \tag{79}$$

²⁵Note that at the corner solution in which the government subsidizes all firms in $(\mu_v^*, \mu]$ through the VC, i.e., when $\mu_1 = \mu$, the objective function given in (69) is slightly different as we indicated in footnote 23 above. In this special case, we solve for the optimal value of μ and the value of the government's objective function at this optimal solution separately. See the problem setup and the objective function in (77).

(iii) The entrepreneurs of firms $i \in (\mu_v^*, \mu_1^*]$ will invest in developing the innovation at date 1 after receiving a subsidy $S_i^* = iC - \alpha p v_0$ channeled through the VC on behalf of the government, where μ_1^* is given by:

$$\mu_1^* = \frac{(1-\lambda)(\delta(1+\alpha\delta) - \lambda(\alpha(2\delta(1+\delta) + 1) - 1))pv_0 + b\delta(1+2\delta)(1-\lambda)^2\mu_v^*}{b\delta(1+2\delta)(1-\lambda)^2 + C(1+\delta)((2\delta-1)\lambda^2 - 4\delta\lambda + \delta)} - \frac{C(1+\delta)\lambda(\delta+\lambda) + \delta(1+2\delta)f(1-\lambda)^2}{b\delta(1+2\delta)(1-\lambda)^2 + C(1+\delta)((2\delta-1)\lambda^2 - 4\delta\lambda + \delta)}. \quad (80)$$

(iv) The entrepreneurs of firms $i \in (\mu_1^*, \mu_{ga}^*]$ will invest in developing the innovation at date 1 after receiving a direct flat subsidy $S^* = \frac{\mu_{ga}^* C - \alpha p v_0}{1-\lambda}$ from the government. The VC will not invest in these firms.

(v) The total fraction of firms that invest in developing the innovation in industry P, μ_{ga}^* , is characterized by

$$\mu_{ga}^* = \frac{pv_0(b\delta(1-\lambda)^2(1+\alpha\delta) + C(\lambda^2(\alpha((\delta-1)\delta-1) + \delta) - \lambda\delta(2+\alpha(1+3\delta)) + \delta(1+\alpha\delta)))}{C(b\delta(1+2\delta)(1-\lambda)^2 + C(1+\delta)((2\delta-1)\lambda^2 - 4\lambda\delta + \delta))} + \frac{b\delta(1-\lambda)(\lambda+\delta)\mu_v^*}{b\delta(1+2\delta)(1-\lambda)^2 + C(1+\delta)((2\delta-1)\lambda^2 - 4\lambda\delta + \delta)} - \frac{\delta(1-\lambda)(\lambda(1+\delta)(C+b) + f(\lambda+\delta))}{b\delta(1+2\delta)(1-\lambda)^2 + C(1+\delta)((2\delta-1)\lambda^2 - 4\lambda\delta + \delta)}. \quad (81)$$

Further, the amount of innovation development in this equilibrium is greater than that in the benchmark equilibrium of Proposition 11, since $\mu_{ga}^* > \mu_{gu}^*$ when $f < f_2$, where μ_{gu}^* is given in (64).

The above equilibrium, in which the government channels subsidies through the VC to at least a certain portion of firms with a cost index above μ_v^* (i.e., $\mu_1^* > \mu_v^*$), exists and improves overall incentives to innovate in industry P over the benchmark equilibrium, if the per-firm fixed fee f that the government pays to the VC is below a certain upper threshold f_2 .²⁶ In other words, the government will prefer to implement the above equilibrium rather than the benchmark equilibrium in Proposition 11 if and only if the fixed fee cost f per firm is less than the threshold f_2 . From (81), one should also note that, similar to μ_1 given in (76), the optimal total fraction μ_{ga}^* of firms that invest in developing the innovation in industry P is also decreasing in the per-firm fixed fee f and the marginal fee b . Thus, the above proposition shows that, as long as the fee charged by the

²⁶ At this threshold, the total fraction of firms that invest in developing the innovation in the above equilibrium is μ_{ga}^* given in (81), and is equal to the same fraction of innovating firms (μ_{gu}^* in (64)) in the benchmark equilibrium, in which the government provides untargeted direct subsidies to firms only. Further, at this threshold, μ_1^* is just equal to the corner-solution value μ_v^* .

VC is not too large, the social planner may have a net benefit from taking advantage of the VC's services so that he can better motivate additional firms to innovate by channeling to some of them efficiently targeted subsidies that depend on firms' cost structure, thereby allowing him to rely less on the use of less efficient untargeted (flat) subsidies.

In the above proposition, one should also note that, for the above equilibrium with an interior optimal solution of $\mu_1^* \in (\mu_v^*, \mu_{ga}^*)$ to exist, the fixed fee parameter f must be greater than the lower threshold f_1 . If f is less than this threshold, the government prefers to implement the problem given in (77), since it obtains a greater expected total surplus by channeling all of its subsidies through the VC rather than utilizing a mix of targeted subsidies channeled through the VC and untargeted direct subsidies. The critical parameter that determines the value of the threshold f_1 is λ , which is the fraction of the direct subsidy S^* captured by non-innovating firms in the interval $(\mu_{ga}^*, 1]$. If $\lambda = 0$, the value of the threshold f_1 is given by:

$$f_1(\lambda = 0) = -\frac{b((1 + \alpha\delta)pv_0 - (1 + \delta)C\mu_v^*)}{(1 + \delta)C} = -\frac{(1 - \alpha)(1 - \beta(1 + \delta))b pv_0}{(1 + \delta)C} \leq 0, \quad (82)$$

which is negative for any positive value of b or at most equal to zero when $b = 0$. On the other hand, if $\lambda > 0$, the lower threshold f_1 can be positive, and numerical simulations of our model show that f_1 is increasing in λ . This means that as long as one of the inefficiencies associated with untargeted subsidies (when costs are unobservable) is sufficiently small, i.e., the value of λ is sufficiently small, the above equilibrium with an interior optimal solution of $\mu_1^* \in (\mu_v^*, \mu_{ga}^*)$ will exist.²⁷ The following corollary shows that, when $\lambda = 0$, the lower threshold f_1 becomes irrelevant, and the government prefers to utilize a mix of targeted subsidies channeled to some firms indirectly through the VC and direct (flat) subsidies to other firms to motivate them to innovate at date 1, as long as the

²⁷Numerical simulations of our model also show that the upper threshold f_2 is also increasing in λ , albeit at a smaller rate than f_1 . This means that if λ is too large, f_1 will exceed f_2 so that the social planner's choice will be between two corner solutions for μ_1 ($\mu_1^* = \mu_v^*$ or $\mu_1^* = \mu$). Note also that the upper threshold f_2 does not depend the marginal fee parameter b while our numerical simulations show that, ceteris paribus, f_1 is decreasing in b . Thus, a higher value of the parameter b allows the above equilibrium in Proposition 12 to exist for a larger range of parameters.

fixed fee parameter f is below an upper threshold f_2 .

Corollary 2 *Let $\beta < \frac{1}{(1+\delta)}$ and $\lambda = 0$. Then, the alternative equilibrium described in Proposition 12 will be sustained if and only if $0 < f < f_2 = \frac{(1-\alpha)(1-\beta(1+\delta))pv_0}{(1+2\delta)}$. In this case, the total fraction of firms that invest in developing the innovation in industry P is given by:*

$$\mu_{ga}^* = \frac{(1+\alpha\delta)(1+\frac{b}{C})}{(1+\delta)+(1+2\delta)\frac{b}{C}} \frac{pv_0}{C} + \frac{\delta b}{(1+\delta)C+(1+2\delta)b} \mu_v^* - \frac{\delta}{(1+\delta)C+(1+2\delta)b} f, \quad (83)$$

where $\mu_{gu}^* < \mu_{ga}^* < \mu_{vg}^* = \frac{(1+\alpha\delta)}{(1+\delta)} \frac{pv_0}{C}$. The optimal value of μ_1 is given by:

$$\mu_1^* = \frac{(1+\alpha\delta)}{(1+\delta)+(1+2\delta)\frac{b}{C}} \frac{pv_0}{C} + \frac{(1+2\delta)b}{(1+\delta)C+(1+2\delta)b} \mu_v^* - \frac{(1+2\delta)}{(1+\delta)C+(1+2\delta)b} f. \quad (84)$$

One should note that when $\lambda = 0$, while the total fraction of innovating firms, μ_{ga}^* is still smaller than that in the equilibrium with observable costs (μ_{vg}^* in Proposition 10), the government is able to increase the extent of innovation development in industry P beyond that in the benchmark equilibrium given in Corollary 1, i.e., $\mu_{ga}^* > \mu_{gu}^*$. This is because, while the indirect provision of targeted subsidies to firms through the VC is costly, it helps the government to significantly reduce the use of direct but untargeted (flat) subsidies, which leave large information rents on the table to subsidized firms and are therefore a less efficient means of motivating firms to innovate.

7 Empirical and Policy Implications

Our model generates a number of empirical and policy implications regarding investments in fundamental innovations and alternative mechanisms for motivating such innovations.

(i) *Underinvestment in Fundamental Innovation:* Our analysis shows that there can be significant underinvestment in fundamental innovation, where the benefit or value generated is very widespread and the ability of the innovator to appropriate this value is low. Further, our analysis indicates that the extent of the above underinvestment will vary depending on the extent of appropriability of the value generated by an innovation: at one extreme is basic science, where the extent of underinvestment can be expected to be very high; at the middle of the spectrum are innovations

the value of which can be partly appropriated by the innovator: e.g., a more efficient battery for storing energy; at the other extreme are innovations for which almost the entire value can be appropriated by the innovator, so that there may be no underinvestment at all. Corresponding to the above underinvestment, our analysis suggests that the need for policy intervention (in the form of subsidies or prizes) from governments or philanthropic institutions (foundations, etc.) will be greatest when the appropriability of an innovation to the innovator is the lowest.²⁸

Another implication of our analysis (though not explicitly modeled) is that, for a given level of appropriability, there will be significant underinvestment in innovations where the monetary value of the innovation (if successfully developed) is lower than its social value: two examples of such innovations are an AIDS vaccine or innovations primarily targeted at economically more vulnerable populations (which reduces their monetary value). This, in turn, suggests a greater need for policy interventions on the part of the government or non-profit organizations to motivate such innovations.²⁹

(ii) Subsidy versus Prize when Innovation Development Costs are Observable: There are two important differences between providing a subsidy versus a prize when innovation development costs are observable. First, when costs are observable, a targeted subsidy can be provided based on the development cost incurred by a firm, whereas a prize for developing an innovation is a “one size fits all” mechanism. Second, subsidies are provided *ex ante* (before the development of the innovation) whereas a prize is awarded to a firm (or firms) which has made a meaningful contribution to developing the innovation successfully. This means that subsidies are much more vulnerable to “skimming” of a portion of the public funds provided to a firm by its employees and managers. In countries in which there are reasonably accurate accounting and legal systems so that the above “skimming” losses associated with subsidies is relatively small, the targeting benefit

²⁸ A similar point has been made in an early paper by Nelson (1959), who suggests that much of basic research should remain in the laboratories of universities, which are funded by either the government or non-profit organizations.

²⁹ An example of a policy intervention in this category is the prizes offered by the Bill and Melinda Gates Foundation for the “Reinvent the Toilet Challenge,” to develop “next generation” toilets to deliver safe and sustainable sanitation to the 2.5 billion people around the world currently lacking this.

of subsidies ensures that this mechanism for motivating innovation dominates a prize mechanism. Consistent with the above implication of our analysis, subsidies seem to be the most common mechanism chosen to motivate innovation when the technology involved in the industry of the innovation is more or less conventional (established) and the major players in the industry are well known, so that the costs that may be incurred by various firms for developing the innovation are known.

(iii) Subsidy versus Prize when Innovation Development Costs are Unobservable: When the costs to be incurred by firms for developing an innovation are unobservable, the main advantage of a subsidy scheme over a prize from the observable case (discussed under (ii) above), namely, the ability to target subsidies at the right firms (capable of developing the innovation) and providing subsidies at the right level (depending on their innovation costs) no longer exists. Further, given that subsidies are provided *ex ante* (before the innovation is developed), a significant fraction of the subsidies may be wasted on firms that are fundamentally incapable for developing the innovation, given their cost structure. This is because, even if the government is able to “claw back” a majority of the subsidies not used in developing the innovation from such firms, there will still be a significant wastage of resources given as subsidies provided to such firms.³⁰ In contrast, in the case of a prize, it is given *ex post*, and only to those firms which contributed to developing the innovation successfully, so that a prize dominates subsidies as the best mechanism for motivating innovation when firms’ innovation costs are unobservable. Consistent with this implication of our model, awarding a prize for successfully developing an innovation seems to be the most widely used mechanism to motivate innovation in situations where the technology involved is completely new and the firms or individuals capable of developing the innovation are unknown.

(iv) The Role of Venture Capitalists in Fostering Innovation: Our analysis shows that venture capital investments in firms play a significant role in spurring innovation. This is because venture

³⁰This wastage of resources will be in addition to any skimming of resources by even firms that are capable of developing the innovation.

capitalists may have investments in firms attempting to develop an innovation (the producer industry in our analysis) as well as in those that are able to benefit from using that innovation (the user industry in our analysis), so that the appropriability of the value generated by the innovation will be greater for such venture capitalists compared to the insiders of any given firm attempting to develop the innovation. One implication here is that firms backed by venture capitalists with investments in both producers and users of innovations may be more innovative than non-venture backed firms (see, e.g., Tian and Wang (2014) for supporting evidence).³¹ A second implication is for the relative innovativeness of firms backed by different types of venture capitalists. Corporate venture capitalists are organized as a division of an established firm (e.g., Intel) and may invest in many start-up firms that may either be potential users of the innovations developed by the VC's parent firm or that are attempting to develop innovations that may be of use to their corporate parent. This contrasts with more traditional "independent" venture capitalists who often specialize in backing a number of firms within a particular industry. The above implies that the appropriability of the value created by an innovation is greater for corporate rather independent venture capitalists so that firms backed by corporate venture capitalists will be more innovative compared to those backed by independent venture capitalists. Evidence supporting this prediction is provided by Chemmanur, Loutskina, and Tian (2014), who document that entrepreneurial firms backed by corporate venture capitalists exhibit greater innovation productivity compared to those backed by independent venture capitalists, in terms of both the quantity as well as the quality of the innovations generated by them.³²

(v) Strategic Alliances and Innovation: Our analysis implies that strategic alliances formed by firms for the purpose of pursuing innovation ("R&D alliances") will have greater incentives to invest

³¹See also González-Urbe (2014), who documents that citations of patents held by a firm increase following VC financing, especially citations in patent applications made by other firms in the same venture capitalists' portfolio. This indicates that VCs generate an externality on innovation by portfolio firms, since the knowledge generated by a given firm in the VC's portfolio is used by other firms in the same VC's portfolio for their innovation development.

³²Broadly speaking, our analysis in section 6 also implies that corporate venture capitalists will fund entrepreneurial firms on better financial terms compared to independent venture capitalists because of their ability to appropriate a larger fraction of the value generated by the innovations developed by their portfolio firms. Evidence supporting this implication is provided by Chemmanur, Loutskina, and Tian (2012).

in various innovations compared to stand-alone firms. This is because firms involved in strategic alliances will be able to appropriate a greater fraction of the value of the innovations developed by such strategic alliances compared to stand-alone firms without such alliances. Further, our analysis suggests that the most productive strategic alliances may be formed between potential developers of an innovation and potential users of that innovation, since such alliances have the ability to allow the developers of the innovation to appropriate the value generated from the innovation to the greatest extent, thus reducing underinvestment in innovation development. Finally, our analysis suggests that venture capitalists have an incentive to spur the formation of such strategic alliances. Evidence supporting the first implication above is provided by Chemmanur, Shen, and Xie (2015) and Li, Qiu, and Wang (2015), who document that R&D oriented strategic alliances spur corporate innovation by the firms involved in the alliance. Evidence consistent with the third prediction above is provided by Lindsey (2008), whose evidence suggests that strategic alliances are more frequent among companies sharing a common venture capital investor, and that these alliances improve the probability of a successful exit for venture-backed firms.

(vi) The Desirability of Government-funded Venture Capitalists: Several countries, namely, Canada, China, Belgium, and some U.S. states have formed government-funded venture capital firms, in which, instead of private financing from limited partners, the investments made by the venture capital firm are sourced (at least partially) from government funds. Such government-funded venture capital has become controversial, since the financial performance of these firms has been mixed: see, e.g., Brander, Du, and Hellmann (2014) for Canadian evidence and Alperovych, Hübner, and Lobet (2015) for Belgian evidence. Our analysis has several implications for government-funded venture capital firms. First, our analysis indicates that government-funded venture capital may be a good substitute for direct subsidies by the government for motivating innovations. This is because, as we show in section 5, subsidies become very inefficient when the costs of the innovating firms are unobservable, since, in this case, the government is unable to select the right targets and also unable to choose the right level of subsidies (based on the innovating

firms' cost structure). In this context, if government-funded venture capital firms are able to develop expertise in better identifying target firms for subsidies and in evaluating their cost structure relative to the government, then government-funded venture capital would be a good alternative to direct government subsidies for motivating innovation, as we show in section 6.

Second, our analysis indicates that government-funded venture capitalists should target their investments toward industries characterized by low levels of appropriability of the value generated by the innovation (for innovating firms). This means that government-funded venture capital should target those firms that would not develop fundamental innovations (with significant social value) on their own or with the help of private venture capitalists (independent or corporate). Third, our analysis suggests that it may be optimal for the government to channel their subsidies for motivating innovation by private sector firms partially through venture capital firms and other financial intermediaries who are investing in such firms, and partially through direct subsidies to the innovating firms.³³

Finally, our analysis implies that the appropriate measure of the performance of government-funded venture capital may not be the financial returns of the fund (i.e., returns to total amount invested) or the financial performance of the firms in which they invest. This is because, as we discussed in the previous implication above, the appropriate strategy of the government is to invest in the higher-cost firms attempting to develop innovations of significant social value, thus increasing the probability of successful development of these innovations. Consequently, even when government-funded venture capital firms operate efficiently and fully accomplish their objective, one would expect their financial returns to be lower than private venture capital investments in the same industry.

³³One example in practice of such channelling of government funds through venture capitalists and other financial intermediaries is the Small Business Investment Company (SBIC) Program administered by the Small Business Administration (SBA) of the U.S. government. This program matches investments by private investors (often at a ratio of 2 to 1) and has provided around \$72 billion in capital since its inception in 1958. Two related programs of the SBA are the Small Business Innovation Research (SBIR) and the Small Technology Transfer Research (STRR) programs meant to support R&D spending by firms, especially in the area of cutting-edge technologies.

8 Conclusion

In this paper, we analyzed the mechanisms through which governments and non-profit agencies (principals) may incentivize the development of fundamental innovations, defined as those innovations that have positive social value net of development costs, but have negative net present values to the innovating firms due to the limited appropriability of their value by such firms. We solved for the principal's optimal choice between subsidy schemes and incentivizing prizes under two alternative assumptions: first, where the producing firms' innovation development costs are observable by the principal; and second, where such costs are unobservable. We then introduced venture capitalists (VCs) into the above setting and analyze how VCs may enhance the efficiency of the principal in motivating fundamental innovations. Our analysis has several implications for how to better incentivize innovation development, and in particular, for how government-funded venture capitalists may enhance innovation.

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Appendix A: Proofs of Propositions

Proof of Proposition 1. Note that $\mu pv_0 - \int_0^\mu iC di = \mu pv_0 - C\frac{\mu^2}{2}$. Thus, differentiating the objective function in (1) with respect to μ yields the following first order necessary condition:

$$pv_0 - C\mu = 0. \quad (\text{A.1})$$

Solving this first order necessary condition for μ , we obtain the first-best fraction of firms attempting to innovate as

$$\mu^* = \frac{pv_0}{C}. \quad (\text{A.2})$$

Since the objective function in (1) is concave in μ and given that $C > pv_0$, this implies that μ^* given in (A.2) is the unique interior global maximum of (1) over $[0, 1]$. Plugging the optimal fraction μ^* from (A.2) back into the objective function (1), the total surplus achieved by the principal in the benchmark equilibrium is given by:

$$U^* = \frac{p^2 v_0^2}{2C}. \quad (\text{A.3})$$

■

Proof of Proposition 2. Let $\mu \in (0, 1)$ be the fraction of firms that invest in developing the innovation in industry P. Then, the expected payoff for a firm that invests in developing the innovation is

$$\mu p \times \frac{\alpha v_0}{\mu} + (1 - \mu p) \times 0 = \alpha p v_0.$$

At the same time, the cost of developing the innovation is iC for firm i , so each firm i will decide whether or not to invest in the project based on the difference between its expected payoff and its cost. If $\alpha p v_0 - iC \geq 0$, firm i will invest. Otherwise if $\alpha p v_0 - iC < 0$, firm i will not invest. So in equilibrium, the marginal firm i is firm $\mu_0 = \frac{\alpha p v_0}{C}$. Firms $i \in [0, \frac{\alpha p v_0}{C}]$ will implement the project, while firms $i \in (\frac{\alpha p v_0}{C}, 1]$ will not implement the project.

The expected net benefit to the rest of the society is $(1 - \alpha)\mu_0 p v_0$. The expected benefit to firm $i \in [0, \frac{\alpha p v_0}{C}]$ is $\mu p \times \frac{\alpha v_0}{\mu} + (1 - \mu p) \times 0 = \alpha p v_0$ and the cost is iC . For firm $i \in (\frac{\alpha p v_0}{C}, 1]$, the net payoff is 0. So, the total surplus is

$$U^0 = (1 - \alpha)\mu_0 p v_0 + \int_0^{\mu_0} (\alpha p v_0 - iC) di + \int_{\mu_0}^1 0 di = \frac{(2\alpha - \alpha^2)p^2 v_0^2}{2C}. \quad (\text{A.4})$$

Alternatively, given that only firms in the interval $[0, \mu_0]$ develop the innovation in equilibrium and $\mu_0 = \frac{\alpha p v_0}{C}$, the total surplus is given by:

$$U^0 = \mu_0 p v_0 - \int_0^{\mu_0} iC di = \mu_0 p v_0 - \frac{\mu_0^2}{2} C = \frac{(2\alpha - \alpha^2)p^2 v_0^2}{2C}. \quad (\text{A.5})$$

Finally note that there is underinvestment in developing the innovation in the competitive equilibrium, i.e., $\mu_0 = \frac{\alpha p v_0}{C} < \mu^* = \frac{p v_0}{C}$, if and only if $\alpha < 1$. ■

Proof of Proposition 3. Note that, given the objective function in (10), providing subsidies to firms in industry P is costly to the principal because of the social (opportunity) cost of public funds due to distortionary taxation (δ) and the skimming costs associated with subsidies (γ). Therefore, for any given fraction μ of firms attempting to develop the innovation where $\mu > \mu_0$, it is optimal for the principal to provide the minimum possible subsidy S_i to any firm $i \in [0, \mu]$ so that its participation constraint in (11) is satisfied. This means that for all firms in the interval $[0, \mu_0]$, the optimal subsidy level is zero, i.e.,

$$S_i^* = 0 \quad \forall i \in [0, \mu_0], \quad (\text{A.6})$$

since these firms already earn positive rents from investing in developing the innovation without any subsidy as $\alpha p v_0 - iC \geq 0$ for all $i \in [0, \mu_0]$. Assume, on the contrary, that $S_j > 0$ for some $j \in [0, \mu_0]$ at the optimal solution. The principal can then always increase his objective function in (10) by decreasing S_j by a small positive number $\epsilon > 0$ without violating any participation constraints, contradicting optimality.

Thus, the proof that $S_i^* = 0$ for all $i \in [0, \mu_0]$ follows by contradiction. Similarly, optimality requires that the participation constraint in (11) is binding for all firms that receive a positive subsidy, i.e., for all firms in the interval $(\mu_0, \mu]$, so that we obtain

$$S_i^* = \frac{iC - \alpha p v_0}{1 - \gamma} \quad \forall i \in (\mu_0, \mu]. \quad (\text{A.7})$$

Substituting the subsidy levels from (A.6) and (A.7) into the objective function in (10), the principal's equivalent problem is given by:

$$\max_{\mu} (1 - \alpha)\mu p v_0 + \int_0^{\mu_0} (\alpha p v_0 - iC) di - \int_{\mu_0}^{\mu} (1 + \delta) \left(\frac{iC - \alpha p v_0}{1 - \gamma} \right) di \quad (\text{A.8})$$

After some algebraic simplifications, this problem is equivalent to

$$\max_{\mu} (1 - \alpha)\mu p v_0 + \frac{(1 + \delta)}{(1 - \gamma)} \left(\alpha \mu p v_0 - \frac{C \mu^2}{2} \right) - \left(\frac{(1 + \delta)}{(1 - \gamma)} - 1 \right) \left(\alpha \mu_0 p v_0 - \frac{C \mu_0^2}{2} \right) \quad (\text{A.9})$$

Differentiating this objective function with respect to μ yields the following first order necessary condition:

$$\left[(1 - \alpha) + \frac{(1 + \delta)}{(1 - \gamma)} \alpha \right] p v_0 - \frac{(1 + \delta)}{(1 - \gamma)} C \mu = 0. \quad (\text{A.10})$$

Solving this first order necessary condition for μ , we obtain the optimal fraction of firms attempting to innovate as

$$\mu_{so}^* = \left[\alpha + \frac{(1 - \gamma)}{(1 + \delta)} (1 - \alpha) \right] \frac{p v_0}{C}, \quad (\text{A.11})$$

which is also given in (12). Since the objective function in (A.9) is concave in μ and given that $C > p v_0$, this implies that μ^* given in (A.11) is the unique interior global maximum of (10) over $[0, 1]$. Plugging the optimal fraction μ_{so}^* from (A.11) back into the objective function (A.9), the total surplus achieved by the principal in the subsidy equilibrium with observable costs is given by:

$$U_{so} = \frac{\left(2\alpha - \alpha^2 + \frac{(1 - \gamma)}{(1 + \delta)} (1 - \alpha)^2 \right) p^2 v_0^2}{2C}. \quad (\text{A.12})$$

Finally, note that

$$\frac{\partial \mu_{so}^*}{\partial \gamma} = \frac{-(1 - \alpha) p v_0}{(1 + \delta) C} < 0, \quad \frac{\partial \mu_{so}^*}{\partial \delta} = -(1 - \alpha) \frac{(1 - \gamma) p v_0}{(1 + \delta)^2 C} < 0. \quad (\text{A.13})$$

■

Proof of Proposition 4. In the optimization problem given in (18), the optimal prize amount Z is determined by the break-even condition of the marginal firm whose cost index is equal to μ . Participation constraints given in (19) and (17) imply that the participation constraint for the marginal firm with $i = \mu$ given in (19) must hold as an equality in equilibrium. This is because the function in the left hand side of both constraints, $\alpha p v_0 - iC + pZ$, is decreasing and continuous in i . Since the participation constraint in (19) must be binding (satisfied as an equality) for the marginal firm with $i = \mu$, the prize amount is given by:

$$Z = \frac{\mu C}{p} - \alpha v_0. \quad (\text{A.14})$$

Substituting the prize amount from (A.14) into the objective function in (18) and performing some algebraic simplifications, the principal's equivalent problem is given by:

$$\max_{\mu} (1 + \alpha \delta) \mu p v_0 - \frac{(1 + 2\delta)}{2} C \mu^2 \quad (\text{A.15})$$

Differentiating this objective function with respect to μ yields the following first order necessary condition:

$$(1 + \alpha\delta)pv_0 - (1 + 2\delta)C\mu = 0. \quad (\text{A.16})$$

Solving this first order necessary condition for μ , we obtain the optimal fraction of firms attempting to innovate as

$$\mu_z^* = \frac{(1 + \alpha\delta)pv_0}{(1 + 2\delta)C}, \quad (\text{A.17})$$

which is also given in (20). Since the objective function in (A.15) is concave in μ and given that $C > pv_0$, this implies that μ_z^* given in (A.17) is the unique interior global maximum of (18) over $[0, 1]$. Substituting the optimal fraction μ_z^* from (A.17) into (A.14), the optimal prize amount is given by:

$$Z^* = \frac{(1 - \alpha(1 + \delta))}{(1 + 2\delta)}v_0. \quad (\text{A.18})$$

Plugging the optimal fraction μ_z^* from (A.17) back into the objective function (A.15), the total surplus achieved by the principal in the prize equilibrium with observable costs is given by:

$$U_{zo} = \frac{(1 + \alpha\delta)^2 p^2 v_0^2}{(1 + 2\delta)2C}. \quad (\text{A.19})$$

Next, note that

$$\frac{\partial \mu_z^*}{\partial \alpha} = \frac{\delta}{(1 + 2\delta)} \frac{pv_0}{C} > 0, \quad \frac{\partial Z^*}{\partial \alpha} = \frac{-(1 + \delta)}{(1 + 2\delta)}v_0 < 0, \quad (\text{A.20})$$

and

$$\frac{\partial \mu_z^*}{\partial \delta} = \frac{(\alpha - 2)}{(1 + 2\delta)^2} \frac{pv_0}{C} < 0, \quad \frac{\partial Z^*}{\partial \delta} = \frac{(\alpha - 2)}{(1 + 2\delta)^2}v_0 < 0. \quad (\text{A.21})$$

Finally, the principal will choose to offer a prize rather than not intervene if and only if $U_{zo} > U^0$, which is equivalent to

$$(1 - \alpha(1 + \delta))^2 > 0. \quad (\text{A.22})$$

Given a positive prize requirement $Z^* > 0$, it follows from (A.18) that the condition in (A.22) is equivalent to the condition that $\alpha(1 + \delta) < 1$. Further, it is straightforward to show that $\mu_z^* > \mu_0$ if and only if $\alpha(1 + \delta) < 1$.

■

Proof of Proposition 5. When costs are observable, the optimal fraction firms that invest in developing the innovation through a subsidy mechanism, μ_{so}^* given in (12), will be greater than the optimal fraction of firms that invest in developing the innovation through a prize mechanism, μ_z^* given in (20) if and only if the following condition holds:

$$\mu_{so}^* = \left[\alpha + \frac{(1 - \gamma)}{(1 + \delta)}(1 - \alpha) \right] \frac{pv_0}{C} \geq \mu_z^* = \frac{(1 + \alpha\delta)pv_0}{(1 + 2\delta)C}. \quad (\text{A.23})$$

After some algebraic simplifications, this condition is equivalent to the following condition:

$$\gamma \leq \bar{\gamma}_1 \equiv \frac{\delta(1 + \alpha\delta)}{(1 - \alpha)(1 + 2\delta)}. \quad (\text{A.24})$$

Next, when costs are observable, the principal will prefer subsidizing firms in industry P to offering a prize if and only the following condition holds:

$$U_{so} = \frac{\left(2\alpha - \alpha^2 + \frac{(1 - \gamma)}{(1 + \delta)}(1 - \alpha)^2 \right) p^2 v_0^2}{2C} \geq U_{zo} = \frac{(1 + \alpha\delta)^2 p^2 v_0^2}{(1 + 2\delta)2C}, \quad (\text{A.25})$$

where U_{so} is given in (14) and U_{zo} is given in (22). After some algebraic simplifications, this condition is

equivalent to the following condition:

$$\gamma \leq \bar{\gamma}_2 = \frac{\delta(1 + 2\alpha\delta - \alpha^2(1 + \delta(3 + \delta)))}{(1 - \alpha)^2(1 + 2\delta)}. \quad (\text{A.26})$$

It is straightforward to show that $\bar{\gamma}_2 > \bar{\gamma}_1$ if and only if $\alpha(1 + \delta) < 1$, which we assume to begin with. Thus, it follows that $\bar{\gamma}_2 > \bar{\gamma}_1$.

From the above steps, it follows that if $0 \leq \gamma \leq \bar{\gamma}_1$, then $\mu_{so}^* \geq \mu_z^*$ and $U_{so} > U_{zo}$. Therefore, the result in part (i) of proposition 5 follows. Next, if $\bar{\gamma}_1 < \gamma \leq \bar{\gamma}_2$, then $\mu_{so}^* < \mu_z^*$ and $U_{so} \geq U_{zo}$. Therefore, the result in part (ii) of proposition 5 follows. Finally, if $\gamma > \bar{\gamma}_2$, then $\mu_z^* > \mu_{so}^*$ and $U_{zo} > U_{so}$. Therefore, the result in part (iii) of proposition 5 follows. ■

Proof of Proposition 6. As we also explained in the text, incentive compatibility constraints given in (26) and (27) imply that the incentive compatibility constraint for the marginal firm with $i = \mu_{su}^*$ given in (26) must hold as an equality in equilibrium. This is because the function in the left hand side of both constraints, $\alpha pv_0 - iC + (1 - \gamma)S$, is decreasing and continuous in i . Therefore, the subsidy amount S is given by:

$$S = \frac{\mu C - \alpha pv_0}{(1 - \lambda)(1 - \gamma)}. \quad (\text{A.27})$$

Substituting S from (A.27) into the objective function in (28), the principal's equivalent objective function is given by:

$$\max_{\mu} (1 + \alpha\delta)\mu pv_0 - \frac{(1 + 2\delta)}{2} C \mu^2 - (1 + \delta) \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) (\mu C - \alpha pv_0) \quad (\text{A.28})$$

Differentiating this objective function with respect to μ yields the following first order necessary condition:

$$(1 + \alpha\delta)pv_0 - (1 + 2\delta)C\mu - (1 + \delta) \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) C = 0. \quad (\text{A.29})$$

Solving this first order necessary condition for μ , we obtain the optimal fraction of firms attempting to innovate as

$$\mu_{su}^* = \frac{(1 + \alpha\delta)pv_0}{(1 + 2\delta)C} - \frac{(1 + \delta)}{(1 + 2\delta)} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right). \quad (\text{A.30})$$

which is also given in (31). Since the objective function in (A.28) is concave in μ and given that $C > pv_0$, this implies that μ_{su}^* given in (A.30) is the unique interior global maximum of (28) over $[0, 1]$. Substituting the optimal fraction μ_{su}^* from (A.30) into (A.27), the optimal subsidy amount is given by:

$$S^* = \frac{(1 - \alpha(1 + \delta))pv_0 - (1 + \delta) \left[\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right] C}{(1 + 2\delta)(1 - \lambda)(1 - \gamma)}. \quad (\text{A.31})$$

Plugging the optimal fraction μ_{su}^* from (A.30) back into the objective function (A.28), the total surplus achieved by the principal in the subsidy equilibrium with unobservable costs is given by:

$$U_{su} = \frac{(1 + \alpha\delta)^2 p^2 v_0^2}{(1 + 2\delta)2C} - \Delta, \quad (\text{A.32})$$

where

$$\Delta = \frac{(1 + \delta)}{(1 + 2\delta)} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) \left((1 - \alpha(1 + \delta))pv_0 - \frac{(1 + \delta)C}{2} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) \right). \quad (\text{A.33})$$

Subtracting U_0 given in (5) from U_{su} in (A.32), we obtain

$$U_{su} - U_0 = \frac{\left(pv_0(1 - \alpha(1 + \delta)) - (1 + \delta)C \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) \right)^2}{2(1 + 2\delta)C}. \quad (\text{A.34})$$

Thus, when costs are unobservable, the principal will choose to subsidize firms in industry P rather than not intervene, i.e., $U_{su} > U_0$, if and only if the difference given in (A.34) is positive. Combined with the requirement that the optimal subsidy amount S^* given in (A.31) is positive, this condition is equivalent to the following condition:

$$(1 - \lambda)(1 - \gamma) > \frac{1}{1 + \frac{(1 - \alpha(1 + \delta))pv_0}{(1 + \delta)C}}, \quad (\text{A.35})$$

which is also given in (34). It is straightforward to show that $\mu_{su}^* > \mu_0$ if and only if $U_{su} > U_0$, which is equivalent to (A.35). Finally, note that

$$\frac{\partial \mu_{su}^*}{\partial \lambda} = -\frac{(1 + \delta)}{(1 + 2\delta)} \frac{1}{(1 - \lambda)^2(1 - \gamma)} < 0, \quad \frac{\partial \mu_{su}^*}{\partial \gamma} = -\frac{(1 + \delta)}{(1 + 2\delta)} \frac{1}{(1 - \lambda)(1 - \gamma)^2} < 0, \quad (\text{A.36})$$

and $\frac{\partial \mu_{su}^*}{\partial \delta} = \frac{1}{(1 + 2\delta)^2} \left((\alpha - 2) \frac{pv_0}{C} + \frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) < 0$. ■

Proof of Proposition 7. In the optimization problem given in (39), the optimal prize amount Z is determined by the break-even condition of the marginal firm whose cost index is equal to μ . Participation constraints given in (40) and (41) imply that the participation constraint for the marginal firm with $i = \mu$ given in (40) must hold as an equality in equilibrium. This is because the function in the left hand side of both constraints, $\alpha pv_0 - iC + pZ$, is decreasing and continuous in i . Since the participation constraint in (40) must be binding (satisfied as an equality) for the marginal firm with $i = \mu$, the prize amount is given by:

$$Z = \frac{\mu C}{p} - \alpha v_0. \quad (\text{A.37})$$

Substituting the prize amount from (A.37) into the objective function in (39) and performing some algebraic simplifications, the principal's equivalent problem is given by:

$$\max_{\mu} (1 + \alpha\delta)\mu pv_0 - \frac{(1 + 2\delta)}{2} C \mu^2 \quad (\text{A.38})$$

Differentiating this objective function with respect to μ yields the following first order necessary condition:

$$(1 + \alpha\delta)pv_0 - (1 + 2\delta)C\mu = 0. \quad (\text{A.39})$$

Solving this first order necessary condition for μ , we obtain the optimal fraction of firms attempting to innovate as

$$\mu_z^* = \frac{(1 + \alpha\delta)pv_0}{(1 + 2\delta)C}, \quad (\text{A.40})$$

which is also given in (42). Since the objective function in (A.38) is concave in μ and given that $C > pv_0$, this implies that μ_z^* given in (A.40) is the unique interior global maximum of (39) over $[0, 1]$. Substituting the optimal fraction μ_z^* from (A.40) into (A.37), the optimal prize amount is given by:

$$Z^* = \frac{(1 - \alpha(1 + \delta))}{(1 + 2\delta)} v_0. \quad (\text{A.41})$$

Plugging the optimal fraction μ_z^* from (A.40) back into the objective function (A.38), the total surplus achieved by the principal in the prize equilibrium with unobservable costs is given by:

$$U_{zu} = \frac{(1 + \alpha\delta)^2 p^2 v_0^2}{(1 + 2\delta) 2C}. \quad (\text{A.42})$$

Next, note that

$$\frac{\partial \mu_z^*}{\partial \alpha} = \frac{\delta}{(1 + 2\delta)} \frac{pv_0}{C} > 0, \quad \frac{\partial Z^*}{\partial \alpha} = \frac{-(1 + \delta)}{(1 + 2\delta)} v_0 < 0, \quad (\text{A.43})$$

and

$$\frac{\partial \mu_z^*}{\partial \delta} = \frac{(\alpha - 2)}{(1 + 2\delta)^2} \frac{pv_0}{C} < 0, \quad \frac{\partial Z^*}{\partial \delta} = \frac{(\alpha - 2)}{(1 + 2\delta)^2} v_0 < 0. \quad (\text{A.44})$$

Finally, the principal will choose to offer a prize rather than not intervene if and only if $U_{zu} > U^0$, which is equivalent to

$$(1 - \alpha(1 + \delta))^2 > 0. \quad (\text{A.45})$$

Given a positive prize requirement $Z^* > 0$, it follows from (A.41) that the condition in (A.45) is equivalent to the condition that $\alpha(1 + \delta) < 1$. Further, it is straightforward to show that $\mu_z^* > \mu_0$ if and only if $\alpha(1 + \delta) < 1$.

■

Proof of Proposition 8. The total surplus achieved by the principal in the subsidy equilibrium with unobservable costs, U_{su} , is given in (A.32). The total surplus achieved by the principal in the prize equilibrium with unobservable costs, U_{zu} , is given in (A.42). It follows that, when costs are not observable, the principal will prefer offering a prize firms in industry P to subsidizing them if and only if $U_{zu} > U_{su}$, which is equivalent to the following condition:

$$U_{zu} = \frac{(1 + \alpha\delta)^2}{(1 + 2\delta)} \frac{p^2 v_0^2}{2C} > U_{su} = \frac{(1 + \alpha\delta)^2}{(1 + 2\delta)} \frac{p^2 v_0^2}{2C} - \Delta, \quad (\text{A.46})$$

where Δ is given in (A.33). Note that, given that condition given in (34) holds, it follows that

$$\Delta = \frac{(1 + \delta)}{(1 + 2\delta)} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) \left((1 - \alpha(1 + \delta))pv_0 - \frac{(1 + \delta)C}{2} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) \right) > 0. \quad (\text{A.47})$$

Since $\Delta > 0$, it follows from (A.46) that $U_{zu} > U_{su}$. Finally, note that

$$\mu_z^* - \mu_{su}^* = \frac{(1 + \delta)}{(1 + 2\delta)} \left(\frac{1}{(1 - \lambda)(1 - \gamma)} - 1 \right) > 0, \quad (\text{A.48})$$

so that the fraction of firms that invest in developing the innovation when the principal uses a prize offering is greater than that when he uses a subsidy scheme, i.e., $\mu_z^* > \mu_{su}^*$, when costs are not observable. ■

Proof of Proposition 9. In the VC's optimization problem given in (48) and (49), we first show that, at the optimal solution, the entrepreneurs' participation constraint in (49) will be binding (satisfied as an equality). The proof is by contradiction. Suppose that at the optimal solution, there exists some $j \in (\mu_0, \mu_v]$ for which the following condition holds:

$$(1 - s_v^j) \alpha p v_0 - \rho j C > 0. \quad (\text{A.49})$$

Then, the VC can increase his equity stake s_v^j by a small amount $\varepsilon > 0$ while still satisfying the entrepreneur's participation constraint in (49) and increasing his expected payoff in (48). This contradicts optimality. Therefore, the entrepreneurs' participation constraint in (49) must be binding at the optimal solution so that the VC's equity stakes in firms with $i \in (\mu_0, \mu_v]$ are given by:

$$s_v^i = \frac{(1 - \rho)iC + (\alpha p v_0 - iC)}{\alpha p v_0}, \quad \forall i \in (\mu_0, \mu_v]. \quad (\text{A.50})$$

Substituting s_v^i from (A.50) into the VC's objective function in (48), we obtain the following equivalent problem:

$$\max_{\mu_v} \int_{\mu_0}^{\mu_v} (\alpha p v_0 - iC) di + \beta(1 - \alpha)\mu_v p v_0, \quad (\text{A.51})$$

which can be further simplified to the following equivalent problem:

$$\max_{\mu_v} (\alpha + \beta(1 - \alpha)) \mu_v p v_0 - \frac{C}{2} \mu_v^2 - \frac{\alpha^2 p^2 v_0^2}{2C}. \quad (\text{A.52})$$

Differentiating this objective function with respect to μ_v yields the following first order necessary condition:

$$(\alpha + \beta(1 - \alpha))pv_0 - C\mu_v = 0. \quad (\text{A.53})$$

Solving this first order necessary condition for μ_v , we obtain the optimal fraction of firms attempting to innovate as

$$\mu_v^* = (\alpha + \beta(1 - \alpha)) \frac{pv_0}{C}, \quad (\text{A.54})$$

which is also given in (50). Since the objective function in (A.52) is concave in μ_v and given that $C > pv_0$, this implies that μ_v^* given in (A.54) is the unique interior global maximum of (48) over $[0, 1]$.

For each firm i that invests in developing the innovation in industry P, i.e., for each firm with cost index $i \in [0, \mu_v^*]$, the firm value is equal to αpv_0 , otherwise it is equal to zero. Since we assumed that entrepreneurs invest a fraction ρ of the required R&D investment iC , and the entire positive NPV of the R&D project, $(\alpha pv_0 - iC)$, is captured by the entrepreneur of a firm with $i \in [0, \mu_0]$, it follows that the entrepreneur of a such firm will retain the following equity stake in her firm:

$$s_e^{i,*} = \frac{\rho iC + (\alpha pv_0 - iC)}{\alpha pv_0}, \quad \forall i \in [0, \mu_0]. \quad (\text{A.55})$$

In these firms, the VC just breaks even by investing a fraction $(1 - \rho)$ of the required investment iC . This means that the VC's net expected payoff from investing $(1 - \rho)iC$ in a firm i with $i \in [0, \mu_0]$ is zero, since she holds an equity claim worth $(1 - \rho)iC$ on the future cash flows of the firm. Therefore, the VC's equilibrium equity stakes in these firms are given by:

$$s_v^{i,*} = \frac{(1 - \rho)iC}{\alpha pv_0}, \quad \forall i \in [0, \mu_0]. \quad (\text{A.56})$$

One should also note that $s_v^{i,*} = 1 - s_e^{i,*}$.

In firms with $i \in (\mu_0, \mu_v^*]$, the NPV of the R&D investment is negative, i.e., $\alpha pv_0 - iC < 0$ for all $i \in (\mu_0, \mu_v^*]$. As we showed in (A.50) above, the VC is willing to receive this negative NPV as his net expected payoff from investing $(1 - \rho)iC$ in each firm with $i \in (\mu_0, \mu_v^*]$ so that his equilibrium equity stakes in these firms are given by:

$$s_e^{i,*} = \frac{(1 - \rho)iC + (\alpha pv_0 - iC)}{\alpha pv_0}, \quad \forall i \in (\mu_0, \mu_v^*]. \quad (\text{A.57})$$

The entrepreneurs of these firms just break even by investing a fraction ρ of the required investment iC . This means that if $i \in (\mu_0, \mu_v^*]$, the firm i 's entrepreneur's net expected payoff from investing ρiC in her firm is zero, since she holds an equity claim worth ρiC on the future cash flows of the firm. Therefore, the entrepreneurs' equilibrium equity stakes in these firms are given by:

$$s_e^{i,*} = \frac{\rho iC}{\alpha pv_0}, \quad \forall i \in (\mu_0, \mu_v^*]. \quad (\text{A.58})$$

By construction, firms with $i \in (\mu_v^*, 1]$ do not invest in developing the innovation in equilibrium. Therefore, $s_v^{i,*} = 0$ for all $i \in (\mu_v^*, 1]$.

Finally, note that $\mu_v^* - \mu_0 = \beta(1 - \alpha) \frac{pv_0}{C} > 0$. ■

Proof of Proposition 10. Given the strategy profile of the proposed Nash equilibrium, we will show that, for each player of the game, it is optimal to pursue his or her equilibrium strategy given the equilibrium strategies of the other players.

The entrepreneur of a firm with $i \in [0, \mu_0]$ finds it optimal to invest ρiC to develop the innovation, because she captures the entire positive NPV of the R&D project, which is equal to $(\alpha pv_0 - iC)$, through her equity stake

$$s_e^{i,*} = 1 - s_v^{i,*} = \frac{\rho iC + (\alpha pv_0 - iC)}{\alpha pv_0}, \quad (\text{A.59})$$

where the VC's equilibrium equity stake $s_v^{i,*}$ is given in (55). Given the equilibrium strategies of other players, the VC also has no incentive to deviate from his equilibrium strategy of investing $(1 - \rho)iC$ in a

firm with $i \in [0, \mu_0]$, because his expected payoff is zero given his equity stake $s_v^{i,*}$ given in (55). Clearly, the government has no incentive to subsidize a firm with $i \in [0, \mu_0]$ to motivate it to innovate given the equilibrium strategies of the entrepreneurs and the VC.

Next, the entrepreneur of a firm with $i \in (\mu_0, \mu_v^*]$ has no incentive to deviate from her equilibrium strategy of investing ρiC into her firm's R&D project, because her net expected payoff from this investment is zero (she just breaks even) given her equity stake

$$s_e^{i,*} = 1 - s_v^{i,*} = \frac{\rho iC}{\alpha p v_0}, \quad (\text{A.60})$$

where the VC's equilibrium equity stake $s_v^{i,*}$ is given in (56). When determining his optimal investment strategy with regard to firms with $i \in (\mu_0, \mu_v^*]$, the VC solves the following problem given the equilibrium strategies of other players:

$$\max_{\mu \geq \mu_0, s_v^i} \int_{\mu_0}^{\mu} (-(1-\rho)iC + s_v^i(\alpha p v_0)) di + \beta(1-\alpha)(\mu + (\mu_{vg}^* - \mu_v^*)) p v_0 \quad (\text{A.61})$$

$$\text{s.t. } (1 - s_v^i) \alpha p v_0 - \rho iC = 0, \quad \forall i \in (\mu_0, \mu]. \quad (\text{A.62})$$

Since the entrepreneur's equilibrium equity stake is given in (A.60) in this case, the VC's equilibrium equity stake is already identified by the entrepreneurs' binding participation constraints in (A.62):

$$s_v^{i,*} = \frac{(1-\rho)iC + (\alpha p v_0 - iC)}{\alpha p v_0}, \quad \forall i \in (\mu_0, \mu]. \quad (\text{A.63})$$

Substituting $s_v^{i,*}$ into the VC's objective function in (A.61), we obtain the following equivalent problem:

$$\max_{\mu \geq \mu_0} \int_{\mu_0}^{\mu} (\alpha p v_0 - iC) di + \beta(1-\alpha)(\mu + (\mu_{vg}^* - \mu_v^*)) p v_0, \quad (\text{A.64})$$

which can be further simplified to the following equivalent problem:

$$\max_{\mu \geq \mu_0} (\alpha + \beta(1-\alpha))\mu p v_0 - \frac{C}{2}\mu^2 - \frac{\alpha^2 p^2 v_0^2}{2C} + \beta(1-\alpha)(\mu_{vg}^* - \mu_v^*) p v_0. \quad (\text{A.65})$$

Differentiating this objective function with respect to μ yields the following first order necessary condition:

$$(\alpha + \beta(1-\alpha)) p v_0 - C\mu = 0. \quad (\text{A.66})$$

Solving this first order necessary condition for μ , we obtain the optimal fraction of firms attempting to innovate as

$$\mu^* = (\alpha + \beta(1-\alpha)) \frac{p v_0}{C}, \quad (\text{A.67})$$

which is equal to μ_v^* given in (50). Since the objective function in (A.65) is concave in μ and given that $C > p v_0$, this implies that μ_v^* is the unique interior global maximum of (A.61) over $[\mu_0, 1]$. Hence, as outlined in part (ii) of Proposition 10, given the equilibrium strategies of other players, it is optimal for the VC to invest $(1-\rho)iC$ into the R&D projects of firms with $i \in (\mu_0, \mu_v^*]$ in exchange for a fraction $s_v^{i,*}$ (given in (56)) of firm i 's equity. Note also that the government has no incentive to subsidize a firm with $i \in (\mu_0, \mu_v^*]$ to motivate it to innovate given the equilibrium strategies of the entrepreneurs and the VC.

Finally, consider firms with $i > \mu_v^*$. Given the equilibrium strategies of other players, the entrepreneur of firm i , where $i \in (\mu_v^*, \mu_{vg}^*]$, has no incentive to deviate from her equilibrium strategy of investing iC into the firm's R&D project with expected cash flows worth $\alpha p v_0$ (and thereby owning the entire equity of the firm) and just break even after receiving a subsidy $S_i^* = iC - \alpha p v_0$ from the government. As we showed in the optimal solution to the VC's problem given in (A.61), the VC has no incentive to invest in any firm with $i > \mu_v^*$ given the equilibrium strategies of other players. Next, given the equilibrium strategies of the

entrepreneurs and the VC, the government solves the following problem:

$$\max_{\mu, S_i} (1 - \alpha)\mu pv_0 + \int_0^{\mu_v^*} (\alpha pv_0 - iC) di + \int_{\mu_v^*}^{\mu} (\alpha pv_0 - iC + S_i) di - \int_{\mu_v^*}^{\mu} (1 + \delta) S_i di \quad (\text{A.68})$$

$$\text{s.t. } \alpha pv_0 - iC + S_i \geq 0 \quad \forall i \in (\mu_v^*, \mu]. \quad (\text{A.69})$$

Note that due to the social cost of public funds ($\delta > 0$), the participation constraint (A.69) of the entrepreneur of firm i , where $i \in (\mu_v^*, \mu]$, is binding at the optimal solution. This is already reflected in the equilibrium strategy of the entrepreneur as well. Thus, the subsidy amount is given by:

$$S_i = iC - \alpha pv_0, \quad \forall i \in (\mu_v^*, \mu]. \quad (\text{A.70})$$

Substituting S_i from (A.70) into the objective function in (A.68), we obtain the following equivalent problem:

$$\max_{\mu} (1 - \alpha)\mu pv_0 + \int_0^{\mu} (\alpha pv_0 - iC) di - \int_{\mu_v^*}^{\mu} \delta (iC - \alpha pv_0) di, \quad (\text{A.71})$$

which can be further simplified to the following equivalent problem:

$$\max_{\mu} (1 + \alpha\delta)\mu pv_0 - (1 + \delta)\frac{C}{2}\mu^2 + \delta \left(\frac{C(\mu_v^*)^2}{2} - \alpha\mu_v^* pv_0 \right). \quad (\text{A.72})$$

Differentiating this objective function with respect to μ yields the following first order necessary condition:

$$(1 + \alpha\delta)pv_0 - (1 + \delta)C\mu = 0. \quad (\text{A.73})$$

Solving this first order necessary condition for μ , we obtain the optimal fraction of firms attempting to innovate as

$$\mu_{vg}^* = \left(\alpha + \frac{1}{(1 + \delta)}(1 - \alpha) \right) \frac{pv_0}{C}, \quad (\text{A.74})$$

which is also given in (58). Since the objective function in (A.72) is concave in μ , and given that $C > pv_0$ and $\beta < \frac{1}{(1 + \delta)}$, this implies that μ_{vg}^* is the unique interior global maximum of (A.68) over $[\mu_v^*, 1]$. Hence, as outlined in parts (iii) and (iv) of Proposition 10, given the equilibrium strategies of other players, it is optimal for the government to subsidize firms in industry P with $(\mu_v^*, \mu_{vg}^*]$. ■

Proof of Proposition 11. Given the equilibrium strategies of the VC and the entrepreneurs of firms in industry P, the government solves the problem given in (61). Optimality requires that the IC constraint in (61) is binding at $i = \mu$ so that $S = \frac{\mu C - \alpha pv_0}{1 - \lambda}$. Therefore, the problem given in (61) can be simplified as:

$$\max_{\mu} \mu pv_0 - \frac{C\mu^2}{2} - \frac{(\lambda + \delta)}{(1 - \lambda)}(\mu C - \alpha pv_0)(\mu - \mu_v^*) - \frac{(1 + \delta)\lambda}{(1 - \lambda)}(\mu C - \alpha pv_0)(1 - \mu) \quad (\text{A.75})$$

Differentiating this objective function with respect to μ yields the following first order necessary condition:

$$(1 + \alpha\delta)pv_0 + \frac{(\lambda + \delta)}{(1 - \lambda)}\mu_v^* C - \frac{(1 + \delta)\lambda}{(1 - \lambda)}C - (1 + 2\delta)C\mu = 0. \quad (\text{A.76})$$

Solving this first order necessary condition for μ , we obtain the optimal fraction of firms attempting to innovate as

$$\mu_{gu}^* = \frac{(1 + \alpha\delta)pv_0}{(1 + 2\delta)C} + \frac{(\lambda + \delta)}{(1 - \lambda)(1 + 2\delta)}\mu_v^* - \frac{\lambda(1 + \delta)}{(1 - \lambda)(1 + 2\delta)}, \quad (\text{A.77})$$

which is also given in (64).

Given the government's equilibrium strategy at date 1 and its precommitment not to intervene at date 1 unless the VC invested in all firms in $[0, \mu_v^*]$ at date 0, it is optimal for the VC to invest $(1 - \rho)iC$ at date 0 in each firm with $i \in [0, \mu_v^*]$ with equity stakes given in (62) and (63) respectively, as this allows the VC to

achieve the following maximum expected payoff given other players' equilibrium strategies:

$$\int_{\mu_0}^{\mu_v^*} (\alpha p v_0 - i C) di + \beta(1 - \alpha) (\mu_v^* + (\mu_{gu}^* - \mu_v^*)) p v_0. \quad (\text{A.78})$$

Given the other players' equilibrium strategies, the entrepreneurs of firms with $i \in [0, \mu_0]$ find it optimal to invest $\rho i C$ (at date 0) to develop the innovation by holding an equity stake $s_{eu}^{i,*} = \frac{\rho i C + (\alpha p v_0 - i C)}{\alpha p v_0}$, since they capture the entire positive NPV of the R&D project. Similarly, the entrepreneurs of firms with $i \in (\mu_0, \mu_v^*]$ are willing to invest $\rho i C$ (at date 0) into the R&D project by holding an equity stake $s_{eu}^{i,*} = \frac{\rho i C}{\alpha p v_0}$, since this allows them to just break even. Finally, the entrepreneurs of firms with $i \in (\mu_v^*, \mu_{gu}^*]$ are willing to invest $i C$ (at date 1) to develop the innovation, because this allows them to earn a positive expected payoff after receiving a government subsidy worth $S^* = \frac{\mu_{gu}^* C - \alpha p v_0}{1 - \lambda}$.

Finally, we solve for the particular value of λ at which μ_{gu}^* given in (64) equal to $\mu_v^* = (\alpha + \beta(1 - \alpha)) \frac{p v_0}{C}$:

$$\bar{\lambda} = \frac{(1 + \alpha \delta) p v_0 - (1 + \delta) C \mu_v^*}{(1 + \alpha \delta) p v_0 - (1 + \delta) C (2 \mu_v^* - 1)} = \frac{(1 - \alpha) (1 - \beta(1 + \delta)) p v_0}{(1 + \delta) C + (1 - 2(1 + \delta) \beta(1 - \alpha) - \alpha(2 + \delta)) p v_0}. \quad (\text{A.79})$$

Since μ_{gu}^* is decreasing in λ , it follows that $\mu_{gu}^* > \mu_v^*$ if and only if $\lambda < \bar{\lambda}$. ■

Proof of Corollary 1. The proof follows by setting $\lambda = 0$ and $\mu_v^* = (\alpha + \beta(1 - \alpha)) \frac{p v_0}{C}$ in the expression for μ_{gu}^* in (64) so that

$$\mu_{gu}^* = \frac{(1 + \alpha \delta) p v_0 + \delta C \mu_v^*}{(1 + 2\delta) C} = \frac{(1 + \delta(\alpha(2 - \beta) + \beta)) p v_0}{(1 + 2\delta) C}. \quad (\text{A.80})$$

Then, it follows that

$$\mu_{gu}^* - \mu_v^* = \frac{(1 + \alpha \delta) p v_0 - (1 + \delta) C \mu_v^*}{(1 + 2\delta) C} = \frac{(1 - \alpha)(1 - (1 + \delta)\beta) p v_0}{(1 + 2\delta) C} > 0, \quad (\text{A.81})$$

since $\beta < \frac{1}{(1 + \delta)}$. Finally, given that $\mu_{vg}^* = \frac{(1 + \alpha \delta) p v_0}{(1 + \delta) C}$ and $\beta < \frac{1}{(1 + \delta)}$, it also follows that

$$\mu_{vg}^* - \mu_{gu}^* = \frac{\delta((1 + \alpha \delta) p v_0 - (1 + \delta) C \mu_v^*)}{(1 + \delta)(1 + 2\delta) C} = \frac{\delta(1 - \alpha)(1 - (1 + \delta)\beta) p v_0}{(1 + \delta)(1 + 2\delta) C} > 0. \quad (\text{A.82})$$

■

Proof of Proposition 12. Given the equilibrium strategies of the VC and the entrepreneurs of firms in industry P and given its new policy tool of subsidizing innovating firms through the VC for a fee, the government compares its expected payoffs at the optimal solutions to two problems it solves at date 1: the problem given in (69) to (72) and the problem given in (77), respectively.

In the first problem, it is straightforward to show that optimality requires that the IR constraint in (70) is binding for all $i \in (\mu_v^*, \mu_1]$ and the IC constraint in (71) is binding at $i = \mu$. Hence, it follows that $S_i = i C - \alpha p v_0$ for all $i \in (\mu_v^*, \mu_1]$ and $S = \frac{\mu C - \alpha p v_0}{(1 - \lambda)}$. Assuming that the optimal value of μ_1 for a given value of μ , given in (76), is in the interior of (μ_v^*, μ) (we will show the conditions under which this occurs below), the problem in (69) to (72) can be simplified as

$$\begin{aligned} \max_{\mu} \quad & \mu p v_0 - \frac{C \mu^2}{2} - \frac{(\lambda + \delta)}{(1 - \lambda)} (\mu C - \alpha p v_0) (\mu - \mu_1) - \frac{(1 + \delta) \lambda}{(1 - \lambda)} (\mu C - \alpha p v_0) (1 - \mu) \\ & - \delta \left(\frac{(C + b) (\mu_1^2 - \mu_v^{*2})}{2} - \alpha p v_0 (\mu_1 - \mu_v^*) + f (\mu_1 - \mu_v^*) - b \mu_v^* (\mu_1 - \mu_v^*) \right) \end{aligned} \quad (\text{A.83})$$

$$\text{s.t.} \quad \mu_1 = \frac{b}{(C + b)} \mu_v^* + \left(\frac{1 + \frac{\lambda}{\delta}}{1 - \lambda} \right) \frac{C}{(C + b)} \mu + \left(1 - \frac{1 + \frac{\lambda}{\delta}}{1 - \lambda} \right) \frac{\alpha p v_0}{C} - \frac{f}{(C + b)}. \quad (\text{A.84})$$

Substituting μ_1 from (A.84) into the objective function in (A.83) and solving for the first order condition of the resulting problem yields the optimal value of μ (i.e., μ_{ga}^*), which is given in (81). Next, substituting μ_{ga}^* from (81) for μ in (76), we obtain the closed-form optimal solution for the cut-off point μ_1 (i.e., μ_1^*), which

is given in (80). Given the value of μ_1^* in (80), it is straightforward to algebraically verify that $\mu_1^* > \mu_v^*$ if and only if the following condition holds:

$$f < f_2 \equiv \frac{(1-\lambda)pv_0(\delta(1+\alpha\delta) + \lambda(1-\alpha(1+2\delta(1+\delta)))) - (1+\delta)C(\lambda(\delta+\lambda) + \mu_v^*((2\delta-1)\lambda^2 - 4\delta\lambda + \delta))}{\delta(1+2\delta)(1-\lambda)^2}. \quad (\text{A.85})$$

We denote the value function of the above problem, i.e., the value of the objective function given in (A.83) at the optimal solution $\mu = \mu_{ga}^*$, by $J(\mu_{ga}^*)$.

In the second problem given in (77), optimality also requires that the IR constraint is binding for all $i \in (\mu_v^*, \mu]$. Hence, it follows that $S_i = iC - \alpha pv_0$ for all $i \in (\mu_v^*, \mu]$. Thus, the problem in (77) can be simplified as

$$\max_{\mu} \quad \mu pv_0 - \frac{C\mu^2}{2} - \delta \left(\frac{(C+b)(\mu^2 - \mu_v^{*2})}{2} - \alpha pv_0(\mu - \mu_v^*) + f(\mu - \mu_v^*) - b\mu_v^*(\mu - \mu_v^*) \right) \quad (\text{A.86})$$

Solving for the first order condition of this problem (differentiating the objective function with respect to μ and setting the resulting derivative equal to zero) yields the following optimal solution:

$$\mu_c^* = \frac{(1+\alpha\delta)pv_0 + \delta b\mu_v^* - \delta f}{(1+\delta)C + \delta b}. \quad (\text{A.87})$$

We denote the value function of the above problem, i.e., the value of the objective function given in (A.86) at the optimal solution $\mu = \mu_c^*$, by $J(\mu_c^*)$.

The threshold value f_1 is defined by the equality $J(\mu_{ga}^*) = J(\mu_c^*)$, which is equivalent to the following equation after some algebraic simplifications:

$$\delta \left(\frac{(C+b)(\mu_c^{*2} - \mu_1^{*2})}{2} - (\alpha pv_0 + b * \mu_v^* - f)(\mu_c^* - \mu_1^*) \right) = ((\lambda + \delta)(\mu_{ga}^* - \mu_1^*) + \lambda(1+\delta)(1 - \mu_{ga}^*)) \frac{(\mu_{ga}^*C - \alpha pv_0)}{(1-\lambda)} + \frac{(\mu_{ga}^{*2} - \mu_c^{*2})C}{2} - (\mu_{ga}^* - \mu_c^*)pv_0, \quad (\text{A.88})$$

where μ_{ga}^* , μ_1^* , and μ_c^* are given in equations (81), (80), and (A.87), respectively. If $\lambda > 0$, the above equation can be simplified to an equivalent quadratic equation in f . The positive root of this quadratic equation is equal to the threshold f_1 . Clearly, the value function $J(\mu_c^*)$ is decreasing in f at a faster rate than the value function $J(\mu_{ga}^*)$. Hence, it follows that $J(\mu_{ga}^*) \geq J(\mu_c^*)$ if and only if $f \geq f_1$. Due to space limitations, we do not give the closed-form expression for f_1 here. One should note if $f > f_1$, it also follows that $\mu_1^* < \mu_{ga}^*$ in the problem given in (69) to (72). This result is obtained because of the discontinuity of the objective function in (69) at $\mu_1 = \mu$, where the term $(1+\delta) \left(\int_{\mu}^1 \lambda S di \right)$ suddenly drops out of the social planner's objective function even though it is positive when $\lambda > 0$ and $\mu < 1$.

In summary, we showed that the government prefers to implement the optimal solution to the problem given in (69) to (72) if and only if $f_1 < f < f_2$. Further, it is straightforward to verify that the closed-form solution for μ_{ga}^* in (81) and the closed-form solution for μ_{gu}^* in (65) are equal to each other if and only if $f = f_2$. Note that while μ_{gu}^* does not depend on f , μ_{ga}^* is decreasing in f since its partial derivative with respect to f is negative:

$$\frac{\partial \mu_{ga}^*}{\partial f} = - \frac{\delta(1-\lambda)(\lambda + \delta)}{b\delta(1+2\delta)(1-\lambda)^2 + C(1+\delta)((2\delta-1)\lambda^2 - 4\lambda\delta + \delta)} < 0. \quad (\text{A.89})$$

Thus, it follows that $\mu_{ga}^* > \mu_{gu}^*$ if $f < f_2$.

Finally, the proof of how the VC and the entrepreneurs of firms in industry P optimally respond to other players' equilibrium strategies mirrors the corresponding part of the proof of Proposition 11 above. Note that the VC obtains a higher expected payoff in this alternative equilibrium compared to his expected payoff in the benchmark equilibrium. This is because $\mu_{ga}^* > \mu_{gu}^*$, from which the VC benefits through his equity holdings in the user industry (β), and the VC earns additional fee income by helping the government subsidize firms in the interval $(\mu_v^*, \mu_1^*]$. Note also that the entrepreneurs of firms with $i \in (\mu_v^*, \mu_1^*]$ are willing to invest iC to develop the innovation, since their targeted subsidies $S_i^* = iC - \alpha pv_0$ are set such that they

just break even by doing so. ■

Proof of Corollary 2. The proof follows by setting $\lambda = 0$ in the expressions for μ_{ga}^* in (65) and for μ_1^* in (80). Then, it follows that if $\lambda = 0$, μ_{ga}^* and μ_1^* are given by (83) and (84), respectively. By substituting $\lambda = 0$ in the expression for the threshold f_2 in (A.85), we also obtain

$$f_2 = \frac{(1 + \alpha\delta)pv_0 - (1 + \delta)C\mu_v^*}{(1 + 2\delta)}. \quad (\text{A.90})$$

After substituting $\mu_v^* = (\alpha + \beta(1 - \alpha))\frac{pv_0}{C}$, this further simplifies to

$$f_2 = \frac{(1 - \alpha)(1 - \beta(1 + \delta))pv_0}{1 + 2\delta}. \quad (\text{A.91})$$

If $\lambda = 0$, the equation (A.88) defining the threshold f_1 above is simplified to the following equivalent equation:

$$\frac{\delta((1 + \delta)C(f - b\mu_v^*) + (1 + \alpha\delta)bpv_0)^2}{2C((1 + 2\delta)b + (1 + \delta)C)(\delta b + (1 + \delta)C)} = 0. \quad (\text{A.92})$$

Solving this equation for f yields

$$f_1 = -\frac{b((1 + \alpha\delta)pv_0 - (1 + \delta)C\mu_v^*)}{(1 + \delta)C} < 0. \quad (\text{A.93})$$

Since $f > 0 > f_1$, it follows that $J(\mu_{ga}^*) > J(\mu_c^*)$ when $\lambda = 0$.

Note also that if $\lambda = 0$, it follows that

$$\mu_{vg}^* - \mu_{ga}^* = \frac{\delta((1 + \delta)C(f - b\mu_v^*) + (1 + \alpha\delta)bpv_0)}{(1 + \delta)C((1 + 2\delta)b + (1 + \delta)C)}, \quad (\text{A.94})$$

where $\mu_{vg}^* = \frac{(1 + \alpha\delta)pv_0}{(1 + \delta)C}$ from (58) and μ_{ga}^* is given in (83). This difference is positive since

$$-(1 + \delta)Cb\mu_v^* + (1 + \alpha\delta)bpv_0 > 0, \quad (\text{A.95})$$

as $\mu_{vg}^* > \mu_v^*$. Finally, if $\lambda = 0$, note that

$$\mu_{ga}^* - \mu_{gu}^* = -\frac{\delta((1 + \delta)C\mu_v^* + (1 + 2\delta)f - (1 + \alpha\delta)pv_0)}{(1 + 2\delta)((1 + 2\delta)b + (1 + \delta)C)}, \quad (\text{A.96})$$

where $\mu_{gu}^* = \frac{(1 + \alpha\delta)pv_0 + \delta C\mu_v^*}{(1 + 2\delta)C}$. Note that this difference is positive if and only if

$$f < f_2 = \frac{(1 - \alpha)(1 - \beta(1 + \delta))pv_0}{1 + 2\delta}. \quad (\text{A.97})$$

■

Appendix B: The Probability of a Successful Innovation and Expected Payoffs

In our model, a successful innovation occurs with probability μp as the outcome of the collective efforts all firms which invested in developing the innovation, and these firms comprise a fraction μ of all firms in the producer industry P. Our model assumes that these innovating firms are either collectively successful in developing the innovation (with probability μp) and share the value αv_0 of the innovation (to their industry) among each other equally, or they are collectively unsuccessful (with probability $(1 - \mu p)$) and they all receive a zero payoff. Since the payoff of an individual firm is $\frac{\alpha v_0}{\mu}$ when an innovation is developed successfully with probability μp , it follows that the expected payoff of an individual firm from a successful innovation is equal to the product of the innovation value per firm ($\frac{\alpha v_0}{\mu}$) and the probability of a successful innovation (μp).

Our model's assumption that firms can be successful in developing the innovation only collectively and that they share the value of the innovation equally among each other in the case of success may appear to be too strong to the reader at the first blush. However, in this appendix, we will show that, in terms of the expected payoff of an individual firm, a very similar result holds even in an alternative setting in which firms are individually successful or unsuccessful in developing the innovation, independently of each other. Namely, we will show that, in this alternative setting, the expected payoff of an individual firm will also be equal to the product of the innovation value per firm (total innovation value divided by the number of firms, $\frac{\alpha v_0}{N}$) and the probability that at least one firm is successful in developing the innovation. Further, we will show in this appendix that, in terms of expected payoffs, the assumption that an innovation prize Z is won and shared by all firms which invested in developing the innovation is not as strong as it seems either.

Note that in our model, there is a continuum of firms in industry P, uniformly distributed over the interval $[0, 1]$. The total number of firms in industry P is normalized to 1. The probability of a successful innovation, μp , is assumed to be directly proportional to the fraction (or number) μ of firms that invest in developing the innovation, which are uniformly distributed over the continuum in the interval $[0, \mu]$. This means that innovation efforts of these μ firms in industry P are not independent of each other, and that they have an equally weighted contribution toward success in innovation development, from which all firms

in the interval $[0, \mu]$ benefit equally. More formally, let the effort level e_i of each firm i in $[0, 1]$ be given by:

$$e_i = \begin{cases} p & \text{if firm } i \text{ exerts effort by investing } iC, \\ 0 & \text{if firm } i \text{ does not exert effort.} \end{cases} \quad (\text{B.1})$$

We assume that the probability of successful innovation in industry P is equal to the sum of R&D efforts by all firms in industry P. Then, if only firms in the interval $[0, \mu]$ try to develop the innovation, the probability of a successful innovation development in industry P is equal to

$$\Pr(\text{success}) = \int_0^1 e_i di = \left(\int_0^\mu p di + \int_\mu^1 0 di \right) = \int_0^\mu p di = \mu p. \quad (\text{B.2})$$

The aggregate value of the innovation to all the innovation-developing μ firms in industry P is equal to αv_0 in the case of success, and this total payoff is uniformly distributed over the continuum of firms in the interval $[0, \mu]$ so that the expected gross payoff density of an atomistic firm in this interval is equal to

$$\frac{\alpha v_0}{\# \text{ of firms}} \times \Pr(\text{success}) = \frac{\alpha v_0}{\mu} \times \mu p = \alpha v_0 p. \quad (\text{B.3})$$

We are going to show that, a similar result analogous to that given in (B.3) can be obtained if we assume that there exists a discrete number of n firms in industry P, trying to develop the innovation individually and independently of each other. Let the probability of successful innovation development for each of these n firms be equal to p . If at least one firm succeeds in developing the innovation, the total value created by it is v_0 , and successful innovators in industry P capture a fraction α of it. Then, the expected gross payoff to each of the n innovation-developing firms in industry P is

$$p \times \left(\sum_{i=0}^{n-1} \binom{n-1}{i} \times p^i (1-p)^{(n-1-i)} \times \frac{\alpha v_0}{i+1} \right) + (1-p) \times 0. \quad (\text{B.4})$$

The meaning of the above formula is as follows. Without loss of generality, firm 1 in industry P will successfully develop the innovation with probability p , and it will share the market with the other i firms that also successfully develop the innovation, where $i \in \{0, 1, \dots, n-1\}$. The probability that i firms out

of the other $(n - 1)$ firms also successfully develop the innovation is $\binom{n-1}{i} \times p^i (1-p)^{(n-1-i)}$, and when this happens, the payoff to firm 1 is $\frac{\alpha v_0}{i+1}$, since it must share the market with the other i successful firms. With probability $(1-p)$, firm 1 will not successfully develop the innovation, and its payoff will be 0.

If we perform the following algebraic steps, we find that the expected gross payoff to each firm given in (B.4) is further simplified to:

$$\begin{aligned}
p \times \left(\sum_{i=0}^{n-1} \binom{n-1}{i} \times p^i (1-p)^{(n-1-i)} \times \frac{\alpha v_0}{i+1} \right) &= p \left(\sum_{i=0}^{n-1} \frac{(n-1)!}{i!(n-1-i)!} \times p^i (1-p)^{(n-1-i)} \times \frac{\alpha v_0}{i+1} \right), \\
&= p \left(\sum_{i=0}^{n-1} \frac{n!}{(i+1)!(n-(i+1))!} \times p^{i+1} (1-p)^{(n-(i+1))} \times \frac{\alpha v_0}{np} \right), \\
&= \frac{\alpha v_0}{n} \left(\sum_{j=1}^n \frac{n!}{j!(n-j)!} \times p^j (1-p)^{(n-j)} \right), \\
&= \frac{\alpha v_0}{n} \times (1 - (1-p)^n). \tag{B.5}
\end{aligned}$$

Note that in this model with a discrete number of firms, the expected gross payoff to each of the n firms that try to develop the innovation consists of two components in (B.5). The first component $\frac{\alpha v_0}{n}$ is the total value captured by industry P, αv_0 , divided by the number of innovation-developing firms n in the industry. This term is analogous to $\frac{\alpha v_0}{\mu}$ in our continuous model given in (B.3). The second component in (B.5) is

$$(1 - (1-p)^n), \tag{B.6}$$

which is equal to the probability of at least one firm succeeding in innovation development in industry P. This term is analogous to the term μp in (B.3), which is equal to the probability of a successful innovation development in our continuous model. Thus, in this discrete model, we also have

$$\frac{\alpha v_0}{n} \times (1 - (1-p)^n) = \frac{\alpha v_0}{\# \text{ of firms}} \times \Pr(\text{success by at least one firm}). \tag{B.7}$$

While the probability of a successful innovation development is nonlinearly increasing in the number of innovation-developing firms n in the discrete model, it is linearly increasing in the number of innovation-developing firms μ in our continuous model. This renders the mathematical analysis of our model much

more tractable.

The Expected Payoff of Winning a Prize

In our model with a continuum of firms in the interval $[0, 1]$, the prize Z is shared uniformly across all innovation-developing firms in the interval $[0, \mu]$ if the innovation is successfully developed. Thus, in the case of successful innovation development, which occurs with probability μp , the prize won by each atomistic firm in the interval $[0, \mu]$ is equal to $\frac{Z}{\mu}$. Thus, the expected prize payoff of an atomistic firm in this interval is equal to

$$\frac{Z}{\# \text{ of firms}} \times \Pr(\text{success}) = \frac{Z}{\mu} \times \mu p = pZ. \quad (\text{B.8})$$

Now consider again the discrete model in which n firms invest in developing the innovation independently of each other and the probability of success for each firm is p . If one firm in industry P successfully develops the innovation, it will win the entire prize Z . If multiple firms successfully develop the innovation, then the successful firms will share the prize equally among each other. Innovation-developing firms that fail to develop the innovation will have no claims to the prize. In this setting where n firms compete with each other to develop the innovation, the expected prize for each individual firm $i \in \{1, \dots, n\}$ is

$$p \times \left(\sum_{i=0}^{n-1} \binom{n-1}{i} \times p^i (1-p)^{(n-1-i)} \times \frac{Z}{i+1} \right) + (1-p) \times 0. \quad (\text{B.9})$$

This means that, w.l.o.g., firm 1 in industry P will successfully develop the innovation with probability p , and it will share the prize Z with the other i firms that also successfully develop the innovation, where $i \in \{0, 1, \dots, n-1\}$. The probability that i firms out of the other $n-1$ firms also successfully develop the innovation is $\binom{n-1}{i} \times p^i (1-p)^{(n-1-i)}$, and when this happens, the prize won by firm i is $\frac{Z}{i+1}$. With probability $(1-p)$, firm i will not successfully develop the innovation, and the payoff is 0.

If we perform the following algebraic steps, we find that the expected prize to each firm given in (B.9)

is further simplified to:

$$\begin{aligned}
p \times \left(\sum_{i=0}^{n-1} \binom{n-1}{i} \times p^i (1-p)^{(n-1-i)} \times \frac{Z}{i+1} \right) &= p \left(\sum_{i=0}^{n-1} \frac{(n-1)!}{i!(n-1-i)!} \times p^i (1-p)^{(n-1-i)} \times \frac{Z}{i+1} \right), \\
&= p \left(\sum_{i=0}^{n-1} \frac{n!}{(i+1)!(n-(i+1))!} \times p^{i+1} (1-p)^{(n-(i+1))} \times \frac{Z}{np} \right), \\
&= \frac{Z}{n} \left(\sum_{j=1}^n \frac{n!}{j!(n-j)!} \times p^j (1-p)^{(n-j)} \right), \\
&= \frac{Z}{n} \times (1 - (1-p)^n). \tag{B.10}
\end{aligned}$$

Note again that, in this discrete model, we also have

$$\frac{Z}{n} \times (1 - (1-p)^n) = \frac{Z}{\# \text{ of firms}} \times \Pr(\text{success by at least one firm}). \tag{B.11}$$

In other words, as in our continuous model, the expected prize of an individual firm in the discrete model is equal to the product of two terms: (a) the total prize Z divided by the number firms trying to develop the innovation, n , and 2) the probability of successful innovation development, $(1 - (1-p)^n)$.