Will Pulling Out the Rug Help?
Uncertainty about Fannie and Freddie’s Federal Guarantee and the Cost of the Subsidy

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Keywords: Fannie Mae and Freddie Mac, Capital Structure, Subsidy
JEL Code: G2; G23; G24

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1. Introduction

Fannie Mae and Freddie Mac (F&F) are financial intermediaries created by the Congress of the United States to foster liquidity in the housing loan market, and to fund loans to certain groups of borrowers such as homeowners, farmers and students (Appendix A provides background information about the activities of these entities). These two Government Sponsored Enterprises earn a fee when they purchase mortgage loans from diverse financial institutions, and then pool and resell them to investors as financial securities. In addition to the fee business, F&F often hold onto some of their securitized loans to earn interest income. Table 1 provides the annual earnings of Freddie Mac from the two sources of business (fee and investment portfolio interest income) for the period 1990 to 2006.

The securitized assets in the investment portfolio held by F&F are funded for the most part by borrowings via sale of debt. It is widely believed by the lenders of funds to F&F (debt holders) that they will be fully reimbursed by the government were F&F to experience distress and default on their obligations. This belief that debt holders are not likely to face losses in the event of bankruptcy has lead to low bond spreads, increased liquidity in these assets and
a corresponding growth in the size of the F&F debt market. In 2005, the face value of outstanding debt of F&F (Fannie and Freddie) totaled more than $2 trillion (see Table 2). However, the assumption of a federal guarantee of F&F debt, that spurred growth in the F&F debt market, has been recently called to question. In a hearing\(^1\) Alan Greenspan remarked:

“there is a perception that debt holders are guaranteed by the full faith and the credit of the United States government, despite the fact that the debentures which they bought and literally say, as required by the law, that this instrument is not backed by the full faith and credit.”

Subsequently, the former Secretary of the Treasury (John Snow) made the following comments, when asked whether he would use his discretionary ability to bail out F&F debt holders in the event of default\(^2\):

“Some commentators believe that this credit availability reinforces the perception that the Federal government backs the debt obligations of the Enterprises. This perception is false.”

Such pronouncements by regulators and politicians are motivated in part by the potential cost that could be borne by taxpayers were these entities to default.\(^3\) The subject of GSE regulation and its impact on systemic risks was also outlined

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\(^1\) See discussion in Housing and Urban Affairs Committee hearing (July 21, 2005) chaired by Senator Richard Shelby.

\(^2\) See testimony of Secretary John Snow before the U.S. House Financial Services Committee Proposals for Housing F&F Reform on April 13, 2005.

\(^3\) Seiler (2003) documents such instances when there have been public pronouncements about the likelihood of government support, were they to default. The author finds that these pronouncements impact debt and equity prices negatively, as financial market participants reassess the risk of these assets.
in a recent speech by Ben Bernanke, the Chairman of the Federal Reserve. The Chairman argued that “financial safety and soundness of GSEs can be enhanced by creating a clear and credible receivership process that leads debt holders to recognize that they would suffer financial losses should a GSE fail”. Such a regulation is intended to exert market discipline on market participants that would in turn reduce the growth of mortgage portfolios and reduce the expected cost of the subsidy to the government.

The uncertainty about whether the government will bail out debt holders and the possibility of new regulations on receivership raises several questions, important both for policy makers as well as for the claimholders (i.e. for debt and equity holders of F&F), and are not fully addressed in the literature.

- First, will the government stance on voicing uncertainty about the guarantee help reduce the cost of this subsidy to taxpayers? Also, how does it impact the values of GSE debt and equity?
- Second, how and to what extent will a clear receivership process impact the value of the subsidy?

In this paper we answer the two questions posed by providing a simple model for the value of F&F where there is uncertainty about the government guarantee. We show that an increase in the possibility that the government will

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4 Chairman Ben S. Bernanke speech on GSE Portfolios, Systemic Risk, and Affordable Housing Honolulu, Hawaii, March 6, 2007 (www.federalreserve.gov/newsevents).
not subsidize debt holders reduces the earnings of the asset portfolio via a feedback mechanism that in turn increases the cost of the subsidy. Also, receivership rules are equivalent to specifying a fixed subsidy amount that must be consistent with existing prices else there would be an immediate and dramatic realignment of prices of GSE debt and impair the GSE’s ability to function.

A Federal Guarantee of debt, and any uncertainty about it, comes into play only if the firm experiences financial distress. Thus, it is important to characterize the probability that F&F may experience distress and default. From the modeling perspective, our main contribution is that we determine this probability using observable variables of the firm and then evaluate how the uncertainty impacts this probability of distress. Note that our estimate of the value of the guarantee does not include other broader measures of indirect costs if a GSE goes bankrupt- e.g., foregone taxes and other social benefits from the existence of a GSE. While these costs are important, they are not a focus of the issues discussed in this paper.

As noted earlier, F&F engage in two lines of business- First, they earn a fee on the mortgages they buy and then resell to investors, after pooling and securitizing the loans. This constitutes their core fee based business. Second, F&F hold mortgage backed security (MBS) portfolios to accrue the spread (difference) between their low cost of capital and the higher yield of the mortgage portfolio. Table 1 shows that fee income for Freddie Mac was 1.8 billion, before costs, in 2006. The fee income has increased steadily and the
growth rate has not varied substantially (standard deviation of earnings growth is 10%). Also, interest income from the portfolio holdings was around 43 billion in 2006 but its earnings growth has varied considerably over the data period (standard deviation of earnings growth is around 38%).

To assess the impact of the guarantee we first separate out the firm into these two parts – the value of the firm due to the earnings in the fee business and the value due to the earnings of interest income (both of these income flows are observable). The firm value of F&F, if there were no debt financing, is simply equal to the present value of the cash flows from the two sources (in the same vein as Passmore (2005)). There is obviously no role for the government guarantee if the firm has not borrowed any money (when there is no debt).

F&F borrow to finance their business because it affords them certain advantages. As long as these advantages outweigh the costs, F&F are inclined to continue borrowing to run their business. The value of the firms when F&F borrow to finance their business is equal to the sum of -

1. the value of the business without borrowing plus
2. the present value of the tax savings and the increased earnings attributable to debt financing when there is a guarantee, and minus any costs that are result of the debt financing.

In terms of the advantages of using debt in the presence of a guarantee, financing via debt adds value for three reasons. First, interest payments are tax deductible and this makes the financing cheaper relative to equity financing. Even though
F&F are exempt of state and local corporate income taxes and the tax advantage of leverage is lower than that for regular corporations, it still accounts for substantive savings. Second, the federal guarantee makes this avenue a cheaper source of long-term funds for F&F because debt holders demand lower coupons since they are likely to face a lower cost in case the firm was to go bankrupt. Other one-time costs incurred in the issuance of debt are lower in contrast to alternate forms of borrowing. This cheap debt financing allows them to increase the earnings spread on their mortgage portfolios relative to the interest cost of financing. Third, other firms that enter into contracts with F&F to help hedge or protect the mortgage security holdings from interest rate changes provide better terms on such contracts (discussed in more detail later). Investors purchase mortgage pools from a GSE even though such collateralized debt has no equity component in it (see Passmore, Sparks and Ingpen (2002)).

In contrast to the benefits of debt, F&F may want to limit the issuance of debt for a number of reasons. Increases in the amount of debt will increase the possibility that the firm will go bankrupt when it is unable to make these promised interest payments. As F&F hold more investment securities in their portfolio that are financed via debt, the increased risks (credit risk, prepayment

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5 Debt issuance to finance the assets of the firm is beneficial for a number of other reasons. F&F debt is exempt from registration with the Securities and Exchange Commission, thus reducing flotation costs. These bonds are treated as quasi-government securities by most investors because investors perceive these securities to be backed by the government. Some banks are allowed to make unlimited investments in F&F debt securities, and F&F securities are eligible as collateral for public deposits as well as for Treasury tax and loan accounts, which makes them attractive to investors. All these benefits result in a lower funding cost for F&F.
risk and interest rate risk) may make the value of these earnings more volatile. Thus, with more debt on the books, the probability that F&F will not be able to meet their obligations increase, and the tax benefits may not be commensurate with the costs. In sum, the benefit of debt financing are tax benefits, low costs of borrowing and better terms on borrowing and hedging of their assets. The cost is that there could be adverse movements in the value of the portfolio and cause the firm to face financial distress.

Uncertainty about the guarantee impacts the firm’s earnings via two channels. A first avenue is the increased interest cost on any new debt that is sold. Now, the uncertainty impacts the firm value through its impact on the expected losses in bankruptcy when the F&F are not able to pay their interest obligations (bankruptcy costs). When there are larger losses in bankruptcy, the debt holders must be compensated by a larger interest payment. Debt holders demand a higher coupon and the profitability of the firm’s asset portfolio will decrease. However the magnitude of this effect is small, given that in the current scenario, F&F values are high enough to make the overall likelihood of bankruptcy quite low. It is important to note that existing borrowings will be subject to the new costs only when existing debt is refunded and new debt is sold. Many of the borrowings are longer term, and the new risk will be re-priced only when any new bonds are sold. In addition, F&F debt enjoys extra liquidity because market participants perceive it to be a convenient short-term place to park their funds. As a result the low yield in part reflects the extra liquidity of
these bonds. Increased uncertainty of the guarantee may add to the coupon demanded by lenders because this liquidity benefit is also reduced.

A second, and more important, channel by which the increased uncertainty impacts F&F immediately is that it reduces the earnings on its MBS portfolio. As we had noted, F&F enter into contracts on a continual basis with other financial institutions to hedge their portfolio against changes in interest rates caused by macroeconomic shocks. Because F&F are constantly entering into new contracts, these contracts allow for re-evaluation of the risk at short intervals as compared to bankruptcy costs on debt which are revalued only when new debt is sold. The costs of hedging the asset portfolio from interest rate shocks will therefore increase right away as a result of this uncertainty. This happens because an increase in the uncertainty about the federal guarantee is likely to increase the capital (margin) required by banks and financial institutions that enter into hedging arrangements with F&F (see for example Cooper and Mello (1992)). Risk managers at these institutions are likely to reassess the losses that would be incurred in case of default within a short time frame. This is somewhat related the liquidity advantage of debt because sellers of these hedging contracts use F&F debt to protect their overall risk, and their ability to do so is linked to liquidity of the debt. The resultant increases in margin would consequently reduce the earnings of the mortgage portfolio and the overall value of the firm substantially in light of the fact that over 80% of the firms’ earnings are derived from the investment portfolio, and hedging of the portfolio is an important
aspect of its activities. An increase in uncertainty about the guarantee that increases the costs of hedging by 10 basis points can decrease the firm value by 15%.\(^6\) Hence small changes in the uncertainty of the guarantee are likely to dramatically impact the firm value via a decrease in returns on the investment portfolio. This aspect of feedback of the credit risk of a firm on its earnings is more pronounced because the capitalized earnings on the security portfolio contribute to a large proportion of the firm value.

To analyze the feedback of the uncertainty on the value of F&F, our model takes as inputs the cash flows from the two lines of business, the fee business and the investment portfolio business described above. In this second line of business, we include the cash flows that are a result of the fact that the security holdings are funded in large part by borrowings or debt that has an implicit guarantee. We obtain closed formed solutions for the value of the firm, its debt and its equity, and relate these values to firm risk, taxes, bankruptcy costs, among other parameters\(^7\). Our main contribution is that we incorporate the probability that the government may let F&F fail in case one of them files for bankruptcy. This allows us to focus on the two questions posed: the impact of the uncertainty on the value of debt, equity and the cost of the subsidy. Despite the simplicity of the model, it is useful from a practical perspective because of the limited inputs required.

\(^6\) These numbers are approximated from F&F balance sheet information.
\(^7\) The quantitative approach to modeling a firm’s assets and liabilities was pioneered by Black and Scholes (1973), Merton (1974) and extended by Black and Cox (1976), and others.
Researchers argue that the “implicit subsidy” that flows from the guarantee by the federal government produces a surplus of billions of dollars and is directed to F&F shareholders (Lehnert, Passmore and Sherlund (2006))\(^8\). We analytically determine the value of this implicit subsidy and the impact on the subsidy when there is some uncertainty about whether the government would step in, were F&F to default. Our article can be regarded as an extension of the reduced form approach employed by Passmore (2005). While Passmore (2005) directly estimates the value of the implicit guarantee using a discounted cash flow approach, we allow the cash flows to be contingent on the value of the firm. We are able to estimate the funding advantage of F&F, the extent to which F&F would reduce their relative holding of mortgage backed securities in the absence of a government guarantee as well as the extent to which shareholders retain the value of the funding advantage. In related work, Lucas and McDonald (2006) specify the dynamics of the firm assets and liabilities to compute the value of the implicit subsidy (see discussion in Lehnert and Passmore (2006)). This article also related to their general approach but our model is more appropriately characterized as a variant of the models of capital structure presented in the

\(^8\) A clearer example how the implicit government backing works is illustrated with Fannie’s problem of insolvency in the 1980s. In the beginning of such decade, the interest rates peaked and earnings on Fannie’s portfolios weren’t high enough to meet its liabilities. The main reason why Fannie made it through was because banks kept lending it money—based on the idea that the government stood behind Fannie. Thus, if everyone thinks that the government will not let F&Fs fail, the likelihood that these companies will not be subject to market discipline will rise, further generating a moral hazard problem.
corporate finance literature. This model is easily implemented and has the advantage of closed form solutions.

The value of the implicit subsidy obtained in our setting, $121 billion, is similar to that obtained by Passmore (2005) even though it is substantially higher than that obtained in some other studies. Also, the valuation of the implicit subsidy is directly related to the formulation obtained at the outset by Passmore (2005). We determine the extent to which uncertainty about the subsidy may affect the cost of the subsidy to the government. Policy makers argue that the government should clarify the potential misperception about the subsidy to the F&F, and thus stem their growth rates and reduce the potential cost to the taxpayer. Interestingly, an increase in the likelihood of revocation can reduce firm value dramatically that may in turn double the expected costs to tax payers. A more realistic avenue to reduce this cost to taxpayers is to cap the size of the investment portfolio. This is consistent with some recent statements by the Federal Reserve Chairman Ben Bernanke (discussed later in Section 4).

Our model requires five basic inputs, each of which are observable- the earnings flow of the fee business, the earnings flow of the portfolio business, the interest cost of debt, its spread over a comparable treasury bond, and the earnings volatility of the business. The model is consequently transparent and easily understood. From an economic and intuitive standpoint, it provides a convenient starting point for the questions posed.
We also analyze the impact of proposed legislation on receivership rules. Receivership rules would place a restriction on when (in terms of firm value) a GSE will file for bankruptcy, and the manner in which the residual assets of the GSE are partitioned and the amount of cash inflows from the government. Such a regulation will remove any uncertainty about the extent of the guarantee, and debt holders are more likely to price their loans consistent with the details of the receivership process. However, as we show, if the receivership rules allow for substantial reevaluation of risks from their current valuations, it could increase the chance that the government may have to put up the promised cash to bail out debt holders.

The article is organized as follows. Section 2 describes our model. Closed form solutions are derived for debt, equity and firm values when there is uncertainty about the extent of the federal guarantee. Section 3 calculates the value of the implicit subsidy and discusses extensions of the model. Section 4 discusses the impact of receivership regulation. Section 5 addresses the impact of uncertainty of the guarantee on other claimholders and Section 6 concludes.

2. A model of F&F

This section describes a basic model for F&F. This set up allows us to analyze the impact of uncertainty about the federal guarantee on the value of the subsidy (Analyzed in Sections 3 and 4). In doing so, we are able to elicit the
impact of uncertainty about the guarantee on the value of its other claims (debt and equity) as well (Section 5).

We start by defining the value of F&F earnings when the firm uses no debt to finance the firm and consequently there is no role of the guarantee. Following the finance literature, the present value of the earnings flow gives the value of the firm. Using this as a starting point, we then analyze the change in the earnings and firm value when debt is used to finance the firm, and there is uncertainty about the guarantee. Use of debt to finance the firm changes the value of the firm because the firm benefits from tax deductions. At the same time, it is important to determine the cost of debt— the probability that the firm may not be able to meet its obligations and file for bankruptcy. This is especially important in our setting because the purpose of this paper is to evaluate the cost of the federal guarantee that comes into play only if the firm is unable to meet its obligations. We obtain the value of F&F debt, the value of the firm and the value of equity (the residual claim on the assets of a firm).9

2.1 Value of the business when there is no debt

Recall that the value of a business is simply the expected present value of its earnings where the present value is taken at the appropriate risk adjusted rate. Accordingly, we first characterize the earnings of the firm when there is no role for uncertainty in the guarantee, i.e., when the firm has no debt on its books.
Fannie Mae and Freddie Mac operate two independent business lines: (1) a fee based business associated with securitizing mortgages that are sold off to other investors, and (2) a portfolio investment business that involves holding various mortgage backed securities.

**Fee based business**

From Table 1, in the year 2006, Freddie Mac earned approximately $1.8 billion in commissions and fees and incurred administrative costs (average of 22%), tax and related costs (average of 23%) to give net earnings of approximately $1 billion. These earnings are uncertain and vary through time. Equation (1), that defines these earnings changes below, merely says that the earnings $\delta_t$ grow at a certain rate each year ($\mu_t$) but experience shocks each period that are drawn from a normal distribution with volatility $\sigma_t$. This equation is commonly referred to as the process governing the earnings changes (for risk-neutral investors):

$$\frac{d\delta_t}{\delta_t} = \mu_t dt + \sigma_t dz_t$$

(1)

where $\mu_t$ is a constant and refers to the growth rate in the fee business, $\sigma_t$ is that instantaneous volatility of the earnings, $dt$ is the increment in time, $dz_t$ is the instantaneous shock to earnings (increment of a Brownian Motion). From Table 1, the earnings for Freddie Mac have an annualized standard deviation of around

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9This approach is commonly employed in the finance literature (e.g., Leland (1994))
10%. Then, the value of the business is equal to the present value of the cash flows it generates: \[ V_1 = \frac{\delta_1}{r - \mu_1}. \] Thus, when the earnings are $1 Billion and \( r = 6\% \) and with a growth rate of \( \mu_1 = 2\% \), the value of this business works out to \[ V_1 = \frac{1}{0.06 - 0.02} = 25 \text{ billion}. \] \( V_1 \) is also referred to as the unlevered value of this line of business because it assumes no consideration about debt financing. Note that the risk-neutral parameter value for the growth in earnings is the observed rates less a risk adjustment. For example, the interest income growth rate is 15% and has a standard deviation of 38%. Using a price of risk of 0.4 per unit of standard deviation, the risk-neutral growth rate works out to the actual growth rate 17% minus the risk premium 0.4*38% (equals 2%)\(^{10}\).

**Portfolio business**

In addition to this first line of business, a second line of business generates returns by holding a portfolio of securities on its books. The firm is able to generate a revenue stream equal to a proportion of the amount of security inventory on its books. In 2006, Freddie Mac earned approximately 43 billion dollars on its investment portfolio of nearly 900 billion. Then, in the absence of tax benefits of debt (because we are first considering an unlevered firm) and

\(^{10}\) The market price of risk computations are discussed in the literature on the equity risk premium. We use an estimate obtained from the aggregate market returns of 12%, volatility of 20% and risk free rate of 4% to give a market price of risk : \((12-4)/20=0.4.\)
including hedging and administrative costs, the net earnings are around 23 billion dollars a year. The earnings of this business, per dollar of securities held (around \(23/900\) or approximately 2.5% in our case), are governed by the following process under the risk neutral measure:

\[
\frac{d\delta_2}{\delta_2} = \mu_2 dt + \sigma_2 dz_2
\]  

(2)

where \(\mu_2\) is a constant, \(\sigma_2\) is that instantaneous volatility of the earnings and \(dz_2\) is the instantaneous shock to earnings. Then the corresponding value of this part of the business is equal to \(V_2 = \frac{F\delta_2}{r - \mu_2}\) where \(F\) is the face value of securities held. When \(r=6\%\) and with a growth rate of \(\mu_2=2\%,\) the value of this business works out to \(\frac{23}{.06 -.02}\)=\$575 billion. Note that the funding advantage is the post tax (40 to 60 basis points less taxes) and less any hedging costs.

It is important to note that the earnings of the firm from this second line of business depend on its creditworthiness. If the government does not support debt holders, financial institutions that trade with F&F would impose additional margin and other costs when they enter into long term contracts with F&F. Hence \(\delta_2\) (earnings per dollar of securities held) of F&F would be lower if the costs of hedging and managing the mortgage portfolio were to increase. Empirical estimates suggest that the total funding advantage of F&F is approximately 40 to 60 basis points (e.g., Passmore (2005)).
Removing the federal guarantee would increase the hedging costs to this extent so as to incorporate the increased risks borne by corporations that enter into longer term deals (e.g., swaps) with F&F. Later, we assume that the earnings on the mortgage portfolio are equal to $\delta_2$ minus a penalty if there is uncertainty about the guarantee. Our assumption is that the earnings are adjusted for the probability ($p$) that the government will not pay up: $\delta_2 = \bar{\delta} - 0.005p$ where $\bar{\delta} = 0.025$ in this case. In other words, when the government revokes the guarantee, earnings would decline by a maximum of 50 basis points or 0.5% per unit of securities held.

The total value of both businesses

The total value (in the absence of debt) of F&F is then given by the sum of the values of the two business lines. Here the value of the firm at time zero is denoted by:

$$V = V_1 + V_2.$$  \hspace{1cm} (3)

In our case this total firm value works out to 25 billion + 575 billion for a total of $600 billion.

As noted, the returns to the fee business and portfolio of securities are time varying and uncertain. Clearly the investment portfolio will bear substantial interest rate and credit risk, even if some of the risks are hedged by a F&F. Now, the overall growth in total firm value ($V$) is contributed by earnings from both lines of business. Correspondingly, the overall risk of the two lines of
business is dependent on the proportion of earnings from each business and the correlation between the businesses. The correlation is nearly zero for the sample period and so the blended risk of the two businesses is approximated as

$$\sigma^2 = (1 - y)^2 \sigma_1^2 + y^2 \sigma_2^2$$

where $y$ is the proportional value of each business, and is fixed. If $y=0.8$, the overall risk of the portfolio is around 30% per year. This risk gives the instantaneous volatility of the firm’s assets and captures all risks that may cause the value of the business to fluctuate. Thus overall volatility is a blend of the core fee business and the interest rate and liquidity risk of the asset portfolio, among other sources of risk.

2.2 Uncertainty in the guarantee and the role of debt

In the previous section, we outlined the value of the business when there is no debt. Suppose now that the F&F borrow to finance some of their assets. While debt financing adds value because of tax deductibility, there is a possibility that the firm is unable to meet its obligations of interest payments and consequently experience distress. To evaluate the probability of such an occurrence, we first assume that the value of the firm at which this occurs is exogenous (we discuss its computation later). Suppose the firm value and its earnings evolve through time unless the firm value declines and reaches a value $V_B$ when it is unable to pay its coupons on the debt and is in financial distress (see Figure 2).
When the firm is in financial distress, the firm is handed over to debt holders and the residual value is distributed amongst them. Let \(1 - \alpha\), where \(0 < \alpha < 1\), be the fraction of the firm value, \(V_b\), that is lost to bankruptcy costs in case the government does not guarantee the liabilities. For example, if \(1 - \alpha = 0.5\), then half of the value of the assets is lost. This leaves debt holders with the amount \(\alpha V_b\), and equity holders with nothing. These losses may in part be direct legal costs, loss of human capital and other such costs.

Additionally, let \(\alpha^*\), where \(0 < \alpha^* < 1\), to be the amount of firm value, \(V_b\), recovered in case of default when government backs their debt, while \(1 - \alpha^*\) is the proportion lost to bankruptcy costs incurred in this scenario. We set the value of \(\alpha^*\) to be greater than the value of \(\alpha\). For example \(\alpha^*\) could be 0.98. Then, 98% of the firm value is recovered when the government steps in to compensate the debt holders. In both instances F&F file for bankruptcy, though it is expected that if government backs their debt, bondholders will recover a higher level of firm value at that time.

**Assumption:** We assume that there is an exogenous probability \(p\) that the government will not cover the losses to debt holders. This parameter captures the uncertainty about the guarantee.

We first obtain the value of the firm and then derive the value of debt and equity in the subsequent sections.
2.2.1 Value of F&F when the costs and benefits of debt are included

While the firm was valued at $600 billion in our earlier example, equity holders may be able to save on taxes and enhance firm value by borrowing and deducting these interest costs from earnings. We now value the F&F when the firm sells debt to finance its business and there is uncertainty about the guarantee.

Consider debt sold at time zero by equity holders to fund the business that in turn requires the firm to pay a coupon flow $C$ to debt holders each period (later we consider alternate debt policies). As we had discussed in the introduction, issuing debt can increase firm value due to tax deductibility of the interest payments but it increases the potential bankruptcy costs. Bankruptcy costs will depend on the probability that the government will not guarantee F&F debt if the firm faces financial distress. If government fully backs F&F debt then the amount lost due to bankruptcy will be low. On the other hand, if F&F have no guarantee at all from the government, then their bankruptcy costs can reduce substantially the amount of firm value left for bondholders. The total value of the firm therefore depends on the probability of government support in case of financial distress, as well as on the level of asset at which default is triggered. Note that financing using long term debt is consistent with the assumptions of the model used herein.
Remark 1: The value of the firm for F&F with uncertainty about the federal guarantee is given by the sum of the firm value without debt, the tax benefits of debt and minus the costs were the firm to go bankrupt:

\[
FV(V_0) = V_0 + \frac{\pi C}{r} w - [p(1-\alpha) + (1-p)(1-\alpha^*)] V_b (1-w)
\]  

(4)

where \( w = \left[1 - \left(\frac{V_0}{V_b}\right)^x\right], \)

\( x = \frac{1}{\sigma^2} \left[\left(\frac{r - \sigma^2}{2}\right) + \sqrt{\left(\frac{r - \sigma^2}{2}\right)^2 + 2r \sigma^2}\right], \)

\( V_0 = V_1 + V_2, \quad V_1 = \frac{\delta_1}{r - \mu_1}, \quad V_2 = \frac{F \delta_2}{r - \mu_2}, \quad \delta_2 = \delta - 0.005 p. \)

Proof. See Appendix.

The firm value in equation (4) is the value of the firm without considering tax benefits of debt \((V_0)\) plus the tax benefits and minus the bankruptcy costs. In equation (5), the tax benefits equal the tax savings conditional on not defaulting- \(\frac{\pi C}{r} w\). Of these tax benefits, the first term \(\frac{\pi C}{r}\) is the present value of the tax benefits of debt and the second term \(w\) corresponds to the probability that the firm will not go bankrupt and continues to receive these tax benefits. Correspondingly, in equation (5), the bankruptcy costs are the present value of losses incurred in default times the probability of defaulting: \(\left(p(1-\alpha) + (1-p)(1-\alpha^*)\right) (1-w). \) Here the loss in firm value is \((1-\alpha^*)\) with a probability \((1-p)\) when the government guarantee is valid. The loss is larger, \((1-\alpha)\), if the debt is not guaranteed. Also note that the earnings \(\delta_2\) on the second line of business equals a fixed amount minus a penalty that depends on the
probability that the government will not guarantee the debt \( (\delta_2 = \delta - 0.005p) \).

Thus uncertainty about the guarantee impacts the income as well as the bankruptcy costs. The overall impact is not linear, i.e., it is not equivalent to using the expected guarantee probability.

There are several possibilities on how to specify when the firm chooses to file for bankruptcy (the barrier \( L_B \)). Lucas and McDonald (2006) assume a level equal to 70% of the value of the liabilities as the trigger point in some examples. If returns to the asset portfolio is negative, and equity holders need to fund coupon payments to the debt holders, the endogenous bankruptcy barrier is characterized by Goldstein, Ju and Leland (2001) equal to \( \frac{C}{r} \left( \frac{x}{r(x+1)} \right) \). The substantive implications of our results are largely unchanged for alternate levels at which the firm is declared bankrupt. In subsequent examples we assume an endogenous barrier (that works out to 80% of liabilities) even though other conditions could be imposed.

2.2.2. The Value of Debt and Equity of F&F

We now value F&F debt when the firm sells debt to finance its business and there is uncertainty about the guarantee. Debt is sold at time zero by equity holders to fund the business, with the following characteristics- infinite maturity and a constant coupon flow \( C \) to debt holders each period. Then, the price of debt at time 0 is written as the sum of two components- the expected present value of:
(a) coupon flows if the firm value remains above $V_B$ and does not experience distress (with probability $w$),

(b) payouts to debt holders if the firm value crosses $V_B$ and goes bankrupt. (with probability 1-$w$),

Remark 2: The value of debt for a F&F with uncertainty about the federal guarantee is given by

$$D(V_{o}) = \frac{C}{r}w + \left[p\alpha + (1 - p)\alpha^*\right]V_B(1-w)$$

Where $w = \left[1 - \left(\frac{V_0}{V_B}\right)\right]^{-x}$, $x = \frac{1}{\sigma^2}\left[\left(r - \frac{\sigma^2}{2}\right) + \sqrt{\left(r - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$, $V_0 = V_1 + V_2$, $V_1 = \frac{\delta_1}{r - \mu_1}$, $V_2 = \frac{F\delta_2}{r - \mu_2}$, $\delta_2 = \bar{\delta} - 0.005p$.

**Proof.** See Appendix.

In equation (5), the first term is the present value of coupons conditional on not going bankrupt (probability of not doing bankrupt is $w$) and the second term is the present value of the payments if the firm becomes bankrupt. Again, in the second term the debt holders receive the payout $\alpha^*$ with a probability $(1-p)$ when the government guarantee is valid. The payout is lower $\alpha$ if the debt is not guaranteed. Note that the coupon $C$ comprises the coupons on the debt raised to finance the mortgage portfolio plus any additional debt raised to finance the core business of the firm.

Equity value is the total value of the firm minus the value of the firm owned by debt holders:
\[ E(V_0) = F(V_0) - D(V_0) \]  \hspace{2cm} (6)

In the subsequent sections we discuss the implications of the model obtained above. Equation (6) gives the value of equity as a function of firm value and the volatility of firm value, amongst other variables.

A common problem in the implementation of these models is that the volatility of firm value is not observable. In this context, Lucas and McDonald (2006) calibrate their model to observed volatility of equity values and other parameters for F&F. The authors obtain the implied volatility of equity values of Fannie Mae from option prices, and find that these values vary between 16.7 percent and 60 percent over the year 2004. In our setting, the value of the unlevered assets can also be gleaned by using the earnings flows of the firm. For our model, we use our base case numbers with earnings of the first line of business equal to \( \delta_1 = 1, \delta_2 = 0.025, \mu_1 = 0.02, \mu_2 = 0.02, r = 6\%, F = 900 \), the value of the first line of business works out to $25 billion and the second business is worth $575 billion (from equations (1) to (3)). We fix the volatility of the first business at 10\%, the volatility of asset values for the second line of business at 38\%, computed using numbers in Table 1. The bankruptcy trigger is endogenous and the interest cost is \( C = 37 \) billion, obtained from the data for 2006 and the tax rate is \( \tau = 0.2 \). These numbers are based on balance sheet data in Table 1.
3. Uncertainty and value of the subsidy

The value of the implicit subsidy to tax payers has been the focus of much research in the academic literature (see for example CBO studies (2001), Hubbard (2004), Jafﬁe (2003), Lucas and McDonald (2006), Naranjo and Toevs (2002), Passmore (2005), Stiglitz, Orsag and Orsag (2002)). There are several problems encountered in the computation of this liability. In particular the use of complex derivatives by F&F, limited information in their annual report, and the lack of regulatory oversight by the Securities and Exchange Commission make it difficult to assess the value of this implicit subsidy. Our approach provides a simple way to compute the value of the subsidy as the present value of payments conditional on default, when the government chooses to pay. Even though it is a simpliﬁcation of the structure of the F&F, it does provide a useful starting point.

Using the set up in Section 2 the value of the subsidy is the present value of the cost incurred by the government, conditional on default and is evaluated as:

\[
S(V_o) = (1 - p)(1 - \alpha)V_B \left[ \frac{\delta_1}{r - \mu_1} + \frac{F(\bar{\delta} - 0.005p)}{r - \mu_2} \right]^{-x}
\]

where \( x = \frac{1}{\sigma^2} \left[ \left( r - \frac{\sigma^2}{2} \right) + \sqrt{\left( r - \frac{\sigma^2}{2} \right)^2 + 2r\sigma^2} \right] \)
In equation (7) \((1-p)\) is the probability that the government will reimburse bondholders, \((1-\alpha)V_b\) is the amount that is reimbursed, and the last term is the probability that the government will incur these costs. As noted earlier, the equation requires five important inputs, each of which are observable- the earnings flow of the fee business, the earnings flow of the portfolio business, the face value of holdings and the yield on this portfolio and the earnings volatility of the business. Using our base case numbers and the recovery rates are set at \(\alpha = 0.5\) and \(\alpha^* = 0.98\) the value of the implicit guarantee works out to $121 billion. It is important to note that equation (7) is a version of equation (1) of Passmore (2005). The value of the subsidy depends on the fee business, the spread earned on mortgage debt and tax benefits. This estimate is consequently within the range of those computed by Passmore (2005) but is substantially higher than that obtained by Lucas and McDonald (2006). Lucas and McDonald (2006) assume that existing debt (leverage) can be reduced in an orderly fashion as the GSE asset values decline. As a result, a part of the debt is repaid, and the government would not need to subsidize the debt holders to a large extent. Also, the assumed asset volatility values are very low and reflect the relatively benign times when MBS portfolios were relatively less volatile. In practice adjustments to portfolios can be relatively quick and severe and smooth unwinding of assets may not be feasible.

Note that uncertainty in the guarantee impacts the subsidy via the first term of equation (7) and via its impact on earnings on the second line of business.
As \( p \) increases, the value of the subsidy may decrease the earnings
\[
\delta_2 = \bar{\delta} - 0.005p
\]
on the investment portfolio, and increase the value of the subsidy. This occurs because the value of the firm declines, and it makes bankruptcy more likely. For example with a 10 percent increase in the probability of no support, the value of the subsidy increases to 140 billion.

Remark 3: The cost of the Federal Subsidy to taxpayers may increase with uncertainty about the guarantee. The extent of the increase depends on the feedback of the uncertainty on the costs of hedging the asset portfolio.

This remark is consistent with recent comments of Ben Bernanke when he urged Congress to bolster regulation of the mortgage giants and suggested limiting their massive holdings to guard against the perils their debt posed to the overall economy.\(^{11}\) The remark also ties into the ongoing discussion about the extent to which the federal guarantee increases the value of F&F. As we explain below the uncertainty impacts the firm value substantively and thus increases the chance that the government will bear some of the costs of distress.

First consider why the guarantee is useful. In our setting, the federal guarantee reduces the risk of losses to bond holders (\( \alpha^* \), the recovered amount, is larger). Therefore, the guarantee allows the firm to earn a spread between the lower costs of debt financing, relative to the yields on mortgage backed securities. The presence of a government guarantee for F&F reduces the cost of
funds for F&F relative to other similar risk businesses run by other corporations. The reduced bankruptcy costs allows the firm to take on more debt and avail the tax benefits of debt.

Also, F&F are better able to manage and hedge their portfolio holdings because of the reduced risk and increased liquidity of their debt. Therefore, F&F are able to accrue profits over and above what the appropriate risk return tradeoff would warrant by increasing the earnings per dollar of securities held ($\delta_2$).

An increase in uncertainty about whether the government will guarantee the debt increases expected bankruptcy costs, and consequently increases the cost of new borrowing and thus reduces the value of the firm. A second effect of increasing the uncertainty about the guarantee is that the cost of hedging and managing the investment portfolio may increase. This impact feeds into the earnings per dollar ($\delta_2$) of the mortgage portfolio held by the F&F.

Using equation (4), Figure 2 provides a graphical analysis of the value of the firm as a function of $p$, the probability that the government will not pay bond holders at default. We set the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.1$ and $\sigma_2 = 0.38$. If this uncertainty increases, $\delta_2$ (earnings per dollar of securities held) of F&F would be lower. We set the fractional return as a linear function of the uncertainty $\delta_2 = \bar{\delta} - 0.005p$, so that higher uncertainty results in

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increased hedging costs by 50 basis points if the government withdraws its guarantee. The recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$. Figure 2 shows that as the probability of no guarantee increases, the value of the firm decreases. This occurs because of the dual avenues via which the uncertainty effects firm value- the increased probability of going bankrupt as well as reduced earnings on the mortgage portfolio. The decline in the value of the firm from increased bankruptcy costs is of a lesser order. The more significant loss in firm value occurs because of a reduction in the value of the firm from reduced spreads earned on the investment side of the business.

Remark 4: F&F subsidy value increases because the firm value decreases with a higher uncertainty about the guarantee. This occurs because of an increase in bankruptcy costs as well as the reduced profits on its mortgage portfolios.

Fannie Mae and Freddie Mac assume a large proportion of the credit risk and prepayment risk of the United States housing market that is currently valued at over 9 trillion dollars. In the presence of a funding advantage, F&F will optimally increase debt financing in order to maximize the value of the firm. At present, regulation limits the risk taking via a restriction on investments to conventional and conforming mortgages where the size of the loan is limited. This limit excludes Fannie Mae and Freddie Mac from only a small fraction of the market. Also, there is a capital regulation equal to 2.5% of the balance sheet assets and .45% of off-balance sheet assets. This capital requirement is small in
proportion to the amount of debt on the books and is unlikely to significantly impact the bankruptcy barrier or buffer the losses given default. As pointed out, if the government were to take away this guarantee, the expected cost to the government increases. A cap on the size of the portfolio will naturally increase the size of equity relative to debt through time and consequently allow the government to reduce the extent of the subsidy in an orderly manner.

Alternate debt policies and rating downgrades

In our setting, we model a firm where the extent of leverage is fixed. However, it is possible that the proportion of the security holdings are adjusted downwards as the business deteriorates. Suppose there is a firm value between the current value of assets and the bankruptcy barrier at which the firm’s assets are adjusted downwards by a certain predetermined amount. Now, the loss making mortgage portfolios can be sold and the proceeds are used to repurchase a part of the debt. Such a more generalized model can be obtained using the setup in this paper, but may not add much to the analysis. Also, it is possible that a deterioration in asset values leads to a ratings downgrade. This in turn would have exactly the same impact as the increase in uncertainty about the guarantee because it would reduce the margin $\delta_2$ earned on the portfolio and have a multiplicative effect on the firm value.
4. Receivership Rules and Portfolio Restrictions\textsuperscript{12}

As noted earlier, the Chairman of the Federal Reserve has argued for the establishment of a clear and credible GSE receivership process to create market discipline for these companies. Such legislation that would establish (1) a well-defined and mandatory process for placing a GSE in receivership and (2) a method for resolving a GSE once it is placed in receivership. The main implication is that market participants would clearly understand that, once certain conditions arise, regulatory forbearance will be impermissible and a GSE receivership will be established. Importantly, the GSE receivership process would include a mechanism for ensuring that both the shareholders and creditors of a failed GSE will bear financial losses. Only if GSE debt holders are persuaded that the failure of a GSE will subject them to losses will they have an incentive to exert market discipline.

The set up in the previous sections allows us to comment on the impact of such legislation and the manner in which it would impact the value of a GSE as well as the value of the subsidy. The legislation would set the point at which a GSE would enter into receivership – in other words the legislation would set the value of the bankruptcy barrier \( V_B \). The firm value could be specified in terms of accounting variables, e.g., earnings, net income, or other related metrics of solvency. A second aspect of the legislation is that it will also specify the

\textsuperscript{12} We thank Wayne Passmore for suggesting this extension to the paper.
manner in which the residual assets are split between different classes of bondholders. Also, the extent of the subsidy that the government provides will determine the amount recovered in this process ($\alpha$).

Therefore putting forth such legislation eliminates any uncertainty about the guarantee. Hence the receivership process is equivalent to quantifying the exact extent of the subsidy and the costs borne by claimants. If the subsidy is lower than what is consistent with current valuations, it would lead to an immediate realignment of the value of the GSE debt and equity and push the company closer to bankruptcy. This in turn would increase the expected probability of paying whatever subsidy is proposed. Again, it is preferable to cap the portfolio and the desired regulations could be put in place according to a calendar, as and when the size of the fee business has grown to make the proposed regulation viable given the current and expected valuations and income growth.

Figure 3 illustrates the interaction between setting the barrier at which the firm is placed in receivership ($V_g$) versus setting the amount of the subsidy ($\alpha$). A lower subsidy value of alpha (or higher losses in bankruptcy) has a more immediate impact on firm value vs. setting a lower barrier at which the assets are placed in receivership if the bankruptcy barrier is far away. Then, adjusting the barrier is a more appropriate choice. This follows from the fact that a lower barrier is equivalent to reducing the probability of financial distress and if it is already far from the current firm value the incremental change in
probability of financial distress is low. This figure does not include the impact of the feedback of the change in credit risk onto the spread earned.

5. Impact of the uncertainty on the value of Debt and Equity

A convenient outcome of our model is that it is able to provide an assessment about the manner in which the value of equity and debt are likely to change with uncertainty about the guarantee or with receivership rules. In this section we also provide an estimate of the extent to which equity-holders of F&F gain by the federal subsidy.

The potential funding advantage of F&F allows the firm to raise debt financing at a lower relative spread in comparison with other firms with similar risk. This funding advantage and its impact on F&F spreads is analyzed by Ambrose and Warga (2002), and others. On the one hand the F&F are able to raise more funds because of the funding advantage, while on the other hand increased leverage may in turn increase spreads because of increased chance of going bankrupt. Our objective is to understand the impact of the uncertainty about the federal guarantee on F&F debt prices and spreads.

Figure 4 illustrates the value of debt as a function of the uncertainty in the federal guarantee, when the initial value of the first line of business is set at base case numbers in Section 2. The volatility of asset values are as in the preceding example and the fractional return on the second line of business is given
by $\delta_2 = \bar{\delta} - 0.005p$. Note that as $p$ increases, the value of debt decreases. Again, an increase in the uncertainty about the federal guarantee decreases the firm value, and consequently the debt value because of the increased likelihood of going bankrupt. The top line in Figure 4 does not account for increased hedging costs of the mortgage portfolio while the bottom line includes such costs. The margin earned on the investment portfolio is lower when there is a larger possibility that there will be no bail out of the debt holders.

Remark 5: F&F debt values decrease with uncertainty about the guarantee because of an increase in bankruptcy costs as well as the reduced profits on its mortgage portfolios.

The equation for bond prices also allows us to evaluate the spread of bond yields over treasury bonds: $Spread = \frac{C}{D(V_0)} - r$. We can therefore analyze the extent to which the funding advantage translates into reduced spreads in comparison with similar risk entities that do not have such a government guarantee. Using our base case numbers, this spread works out to approximately 69 basis points, quite close to the estimates obtained by Passmore (2005) but higher than those estimated in Nothaft, Pearce and Stevanovic (2002) and Ambrose and Warga (2002).

How much do equity holders benefit?
Figure 5 provides a graphical depiction of equity values and debt values as a function of the uncertainty. Equity values are equal to the value of the firm less the value of debt (from equation 6). Equity values decrease in lock step when debt values decrease. This is so because a decrease in firm value makes equity values lower as well (a common outcome often discussed in the corporate finance literature). Again, the firm value declines because the earnings from the mortgage portfolio impact the overall firm value.

It is important to estimate how the subsidy benefits existing equity holders. Suppose there was no subsidy, the first line of business with earnings $\delta_1 = 1$ billion after taxes would be the mainstay of the business and the firm value is around $25$ billion. This business has a historical volatility of revenue changes of $\sigma_i = 10\%$ per year. Now, if the equity holders lever up the firm to large extent and when the firm is financed with debt, the optimal amount of debt is around $20$ billion and the value of equity is around $9$ billion when the firm has an amount of debt that maximizes the value of the firm. If we were to throw in a subsidy at this point, and the firm was to increase its security holdings of MBSs to the extent observed in Freddie Mac, the value of the firm would increase to $635$ billion and the value of equity increases to $67$ billion (using equations (4) to (6)). Hence, the federal subsidy has allowed equity holders to increase their stake by around $58$ billion in this setting. Thus, a large portion of the subsidy that is provided by the government accrues to equity holders (In section 4 later
we compute the value of the subsidy equal to $121 billion of which $58 billion goes to equity holders).

This set up also allows us to examine the extent to which F&F would hold mortgage portfolios were they to be financed without a government guarantee. As noted by Passmore (2005), if F&F were purely private, they would hold far fewer MBSs on their books. If F&F did not have any funding advantage, the fee business would constitute the core business and any securities held would be based on considerations such as diversification benefits of income or simply a providing an interim parking place for these securities.

5. Conclusions

We analyze the implicit subsidy from the federal government via a model that incorporates the ability of F&F to generate a revenue stream by selling mortgage backed securities, as well as by holding these securities on its books via debt financing. We show how a government pronouncement that increases the uncertainty about the federal guarantee to a small extent can cause F&F values to decline by large amounts. The increase in the likelihood that the government will not subsidize the F&F may increase the expected costs of the subsidy to the government. Thus we argue that a cap on the value of F&F investment portfolios is a more effective mechanism to reduce the growth rate of these entities. Our
model is easily applied to elicit the impact of the implicit subsidy on the values of various financial claims of F&F.

We address the extent to which existing equity holders benefit from the subsidy and provide a convenient framework to address related policy issues. Regulators are concerned with providing a market based mechanism to control the growth of the F&F portfolios. The framework allows us to consider the impact of receivership regulation and its features on the value of Fannie and Freddie and on its claimants. Such receivership regulation is equivalent to fixing the extent of the subsidy and therefore the setting of the parameters must be consistent with existing valuations.
References

2001, Federal subsidies and the housing GSEs, (Congressional Budget Office).
Seiler, R. S., 2003, Market discipline of Fannie Mae and Freddie Mac: How do share prices and debt yield spreads respond to new information?, *OFHEO Working paper*.
Appendix
A. Background on F&F
This section describes the business of Fannie Mae and Freddie Mac. F&F are financial intermediaries created by the Congress of the United States to create liquidity in the housing loan market, and to fund loans to certain groups of borrowers such as homeowners, farmers, and students. Fannie Mae was originally created as a wholly owned government corporation in 1938 and was converted into a F&F in 1968. Freddie Mac was created in 1970 as part of the Federal Home Loan Bank System to purchase mortgages from thrifts. Rather than hold mortgages in its portfolio, Freddie Mac pooled these mortgages and sold them after attaching a guarantee for credit risk.

As noted in the introduction, F&F are hybrids of private corporations and federal entities. The F&F are chartered by a federal statute and are exempt from state and local taxes, registration requirements. The US treasury is authorized to lend $2.25 billion to each of them. Banks are allowed to make unlimited investments in F&F debt securities, and F&F securities are eligible as collateral for public deposits as well as for Treasury tax and loan accounts. Also, F&F are exempt from the provisions of many state investor protection laws.

The low spread on F&F debt coupled with the rapid growth of F&F has focused attention on their impact on the systemic risk of the financial markets. Although the debt securities issued by the F&F explicitly state that they do not carry a federal guarantee, their ties to the federal government convince investors of their ties to the federal government and the low risk of their debt.

Table 1 gives the outstanding debt of Fannie Mae and Freddie Mac. Their combined holding of mortgage backed securities as well as the amount of debt has grown to over or near 2 trillion dollars. Market participants as well as regulators increasingly want to determine how the size of the assets and liabilities is likely to affect the chance that the government may need to bail them out (value of the government subsidy). The uncertainty about the guarantee, addressed in this article, also leads to answers about how the implicit subsidy impacts the value of the firm and that of equity, and if the government ought to fix the potential liability of tax payers at the outset.

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13 We adapt this information from other published descriptions of these entities.
B. Proof for Remark 1 and 2

From Leland (1994), the present value of coupon payments is:

\[ \int_0^\infty e^{-r t} C (1 - F(t, V_0, V_B)) dt = \frac{C}{r} \left( 1 - \left[ \frac{V_0}{V_B} \right]^{-\gamma} \right) \]

where

\[ x = \frac{1}{\sigma^2} \left[ \left( \frac{r - \sigma^2}{2} \right) + \sqrt{\left( \frac{r - \sigma^2}{2} \right)^2 + 2r\sigma^2} \right] \]

\[ F(t, V_0, V_T) \]

is the density of the first passage time of the firm value from \( V_0 \) to \( V_B \).

Also the present value of payoffs were the firm to go bankrupt is given by

\[ \int_0^\infty e^{-r t} \alpha V_B g(t, V_0, V_B) dt \]

and is evaluated as: \( \left[ \alpha V_B \right] \left[ \frac{V_0}{V_B} \right]^{-x} \).

The sum of these payments gives the desired result for Remark 2. Similarly, Firm value is equal to the unlevered firm value \( (V_0) \), plus tax benefits \( \frac{\tau C}{r} \left( 1 - \left[ \frac{V_0}{V_B} \right]^{-\gamma} \right) \), and minus bankruptcy costs with a probability \( \left[ \frac{V_0}{V_B} \right]^{-x} \).
Table 1
Fee Income and Interest Income for Freddie Mac (1990-2006)

Data is obtained from the Bloomberg Database and Freddie Mac’s financial statements. The data covers annual reports for the period 1996-2006. All figures are in millions.

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Table 2
Outstanding Debt and MBS holdings for Fannie Mae and Freddie Mac
Data is obtained from the Bloomberg Database and the Department of Housing and Urban Development’s Office of Federal Housing Enterprise Oversight, the Federal Housing Finance Board and Fannie Mae’s and Freddie Mac’s financial statements.

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Figure 1
Sample paths of possible asset values

This figure illustrates two sample paths. One sample path (in bold) illustrates a firm whose value remains above the bankruptcy trigger level and rating change level. A second path (dashed line) illustrates the asset path of a firm that defaults.
Figure 2

Firm value with uncertainty about the subsidy

This figure illustrates the value of debt (Firm value) as a function of uncertainty about the guarantee ($p$). The earnings of the first line of business is $\delta_1 = 1, \delta_2 = 0.25 - 0.005p$, the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.1, \sigma_2 = 0.38$, and the recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$ and $V_B = 500$. 
Figure 3
The interaction between bankruptcy barrier and recovery
This figure illustrates the value of debt (Firm value) as a function of uncertainty about the guarantee ($p$). The earnings of the first line of business is $\delta_1 = 1$, $\delta_2 = 0.025$, the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.1$, $\sigma_2 = 0.38$. 
Figure 4
Debt value and uncertainty about the subsidy
This figure illustrates the value of debt (Debt value) as a function of uncertainty about the guarantee ($p$). The earnings of the first line of business is $\delta_1 = 1$, $\delta_2 = 0.25 - 0.005p$, the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.1$, $\sigma_2 = 0.38$, and the recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$ and $V_B = 500$. 
Figure 5

Equity value and Debt Values and uncertainty about the subsidy
This figure illustrates the value of debt (DV) and Equity (EV) as a function of uncertainty about the guarantee (p). The earnings of the first line of business is $\delta_1 = 1$, $\delta_2 = 0.025 - 0.005p$, the interest rate $r = 6\%$, volatility of asset values $\sigma_1 = 0.1$, $\sigma_2 = 0.38$, and the recovery rates are set at $\alpha = 0.5$ and $\alpha^* = 0.98$ and $V_b = 500$. 