Thresholds and the Welfare Cost of Inflation

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Abstract

The neoclassical growth model is the benchmark framework for studying the welfare effects of inflation. However, existing work likely produces misleading estimates because it does not include any mechanism for inflation to promote capital accumulation and welfare. Such rigid assumptions do not line up with recent evidence of a Tobin effect for advanced economies like the United States. Based upon empirical results, we present a model in which there are threshold effects from inflation to the reliance on cash. Our analysis makes considerable progress in determining the costs of inflation by acknowledging a Tobin effect occurs at low inflation rates but trying to exploit a tradeoff at high rates distorts the efficiency of the payments system with strong adverse consequences for the capital stock and welfare.

JEL Codes: E41, E52, E31, O42
Keywords: Payments System, Money Demand, Capital Accumulation, Inflation

1 Introduction

In his well-cited remarks on the Great Moderation, Bernanke (2004) states “Lower volatility of inflation makes economic planning easier, and reduces the resources devoted to hedging inflation risks...the high level, variability, and unpredictability of inflation [in the 1970s] profoundly affected decisions regarding financial investments and money holdings.” He also stresses that “monetary policies that brought down and stabilized inflation may have led to stabilizing changes in the economy as well.” Moreover, Goodfriend (1993) contends: “Inflation scares are a concern because higher inflation, if realized, would reduce the efficiency of the payments system, with negative consequences for productivity, employment, and economic growth.”

However, in the past few years, central bankers have been on increased alert due to deflation risk which also has its problems. Numerous statements by the Federal Open Market Committee have lamented “measures of underlying inflation are somewhat low, relative to levels that the Committee judges to be consistent, over the longer run, with its dual mandate.” In this manner, the Committee argues that inflation is problematic if it is also too low. As a result, there is likely to be an intermediate level of inflation that maximizes economic activity – the impact of inflation depends on its relationship to the intermediate rate.

Moreover, recent empirical work indicates that the effects of monetary policy are non-linear. For example, Ahmed and Rogers (2000) find that long-run inflation is associated with a Tobin effect whereby inflation promotes economic activity in the United States. Bullard and Keating (1995) point out that inflation may be positively correlated with

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output in low inflation countries. On the other hand, Fischer (1993) and Barro (1995) observe that inflation is correlated with lower rates of economic growth. In comparison to previous work, other evidence is suggestive of thresholds – Ghosh and Phillips (1998) and Khan and Senhadji (2001) find that the impact of inflation is detrimental after it passes a certain level (generally from 10%).

Virtually all monetary growth models are limited in their ability to capture the complexity of the inflation-output relationship. If one wants to focus on the relationship between inflation and output at low inflation rates, a model with a Tobin effect is utilized. On the other hand, if the motivation is aimed at the negative impact of inflation, monetary growth models with a steady-state reverse-Tobin effect are employed. Models which produce a Tobin effect such as Freeman and Huffman (1991) are generally based on the store-of-value role of money while models such as Stockman (1981) which produce a reverse-Tobin effect are based upon the transactions role of money.

Recently, there has been some progress in developing models with multiple steady-states where the effects of persistent monetary policy differ. In this manner, at least the modeling framework and motivations for money demand are the same. For example, Schreft and Smith (1997) demonstrate that inflation has different long-run effects in advanced and developing countries because banks in developing countries hold large amounts of inflation-financed government debt. Ghossoub and Reed (2010) stress that individuals face greater exposure to liquidity risk in poor than advanced countries.

In related work, Ghossoub, Laosuthi, and Reed (2012) contend that the effects of policy vary across countries according to the degree of concentration of the banking sector. In addition, Ghossoub and Reed (2012) demonstrate that the impact of monetary policy depends on the level of financial development.\(^1\) Regardless of the framework, however, the predicted relationships are monotonic and permanent in contrast to existing empirical research. None of these frameworks generate a transition from one inflation-output relationship to another – that is, inflation does not produce a transition between steady-states where the effects of monetary policy are different.\(^2\)

This paper adopts a simple extension of the neoclassical growth model with a cash-in-advance constraint to address the welfare costs of inflation in the presence of a non-linear relationship between inflation and output. In our framework, the extent of the cash-in-advance constraint on investment depends on how much the inflation rate deviates from an intermediate target rate. In this manner, we incorporate Bernanke and Goodfriend’s observations about the negative impact of price instability on money holdings and the distortionary consequences for the payments system.

In order to further motivate our work, we believe it is insightful to briefly review previous work that incorporates a transactions role for money through a cash-in-advance constraint. A majority of the existing literature postulates that an exogenous parameter, say \(\Gamma\), governs the percentage of investment that must be acquired through money balances as a means of payment. Following the insights of Lucas and Stokey (1983, 1987), the remaining fraction \((1 - \Gamma)\) can be acquired through credit. In this manner, the efficiency of the payments system does not depend on economic conditions.

However, simple regression analysis indicates that the reliance on cash depends on the stance of monetary policy. We measure the economy’s reliance on cash through the ratio of M1 to GDP. At inflation rates below 4%, higher rates of inflation are associated with a lower reliance on cash. However, at inflation rates greater than 6.25%, the ratio is

\(^1\)See also Ghossoub (2012).

\(^2\)Antinolli et al. (2007) is the only exception – in their model, domestic inflation leads to currency substitution towards ‘dollars’ producing disintermediation and a negative effect on local output. Thus, one could argue that their framework is more applicable to small countries than large countries such as the United States.
increasing in the inflation rate. In this manner, we view that there is significant evidence on non-linearities in the inflation-payments relationship.

Based upon these results, our theoretical framework posits that the fraction of capital goods \( \Gamma \) which must be acquired through cash depends on the deviation from an intermediate inflation rate. That is, the percentage depends on the relationship of inflation to an inflation threshold, \( \pi \). This deviation reflects how deviations from “price stability” limit investment and output in the economy.\(^3\) Moreover, investment activity also suffers if inflation is too low. The recent experiences of Japan forcibly demonstrate how liquidity hoarding emerges in environments where inflation is too low. In addition, the extension of credit and financial market activity are constrained. As a result, investment is negatively affected. Consequently, at rates below \( \pi \), increasing the inflation rate may improve conditions by relaxing the severity of the cash-in-advance constraint on economic activity.

None of this means to imply that inflation doesn’t raise the cost of holding money and restrict money demand. In fact, our framework implies that inflation can be particularly costly at high inflation rates. First, it reduces the return to money balances which limits investment. Second, it aggravates the tax on investment that individuals must incur as a result of the distortions imposed on the payments system at high inflation rates. We simply admit that low inflation rates are challenging for the payments system too.

As a result, the effects of inflation on economic activity are non-linear. At low inflation rates, a Tobin effect occurs. By comparison, above the threshold, a reverse-Tobin effect emerges. However, the mechanisms through which each effect takes place revolve around the efficiency of the payments system and the intensity of cash balances required for investment. Thus, the transmission mechanisms in our model are new to the literature. Ignoring such non-linearities would generate misleading inferences regarding the welfare cost of inflation.

The remainder of the paper is as follows. Section 2 presents evidence of an inflation threshold in the reliance on cash. Following from the empirical analysis, Section 3 describes our model and the non-linear effects from inflation to economic activity. Section 4 studies the implications of such non-linearities for the welfare cost of inflation. Section 5 provides some concluding remarks.

### 2 Evidence of an Inflation Threshold in the Reliance on Cash

We begin by presenting some basic empirical evidence in favor of a non-monotonic relationship between inflation and the reliance on cash for transactions. There are a number of ways to attempt to measure the importance of cash. For example, the Nilson Report publishes annual data on the percentage of consumer transactions through cash as a means of payment. However, it is not publicly available. Moreover, as the data only reflect transactions by consumers, it is limited in scope. It is also relatively short which limits the ability to use it for econometric study. There is also the annual Federal Reserve Payments Systems Study which provides information on noncash transactions but it has only recently been implemented. Consequently, we choose to measure the reliance on cash through the ratio of M1 to GDP.

Figure 1 plots the reliance on cash against the inflation rate from 1959-2012. The average reliance on cash over the sample is approximately 15.6%. Interestingly, the data appear to exhibit a non-monotonic relationship. At a reliance that exceeds the mean,\(^3\) Boyd, Levine, and Smith (2001) find that bank credit and financial market activity decline after inflation surpasses a given threshold.
there is evidence of a negative relationship. By comparison, if the reliance on cash is relatively low, the Figure shows that a positive relationship may be observed.

We begin by presenting a simple regression of the reliance on cash against a time trend, inflation-squared, and GDP over the entire sample:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>.2495796</td>
<td>.0034332</td>
<td>0.000</td>
</tr>
<tr>
<td>Time</td>
<td>-.0052424</td>
<td>.0005464</td>
<td>0.000</td>
</tr>
<tr>
<td>GDP</td>
<td>8.55e-6</td>
<td>1.97e-6</td>
<td>0.000</td>
</tr>
<tr>
<td>Inflation$^2$</td>
<td>-1.946119</td>
<td>.4917445</td>
<td>0.000</td>
</tr>
</tbody>
</table>

$R^2 = .9165 \quad Number \ of \ Observations = 54 \quad Table 1: \ Inflation \ and \ the \ Reliance \ on \ Cash \ in \ Full \ Sample$

A number of observations stand out from the results. First, the time trend is negative and highly significant. As pointed out in Schreft and Smith (2000), technological improvements in the payments system have reduced the reliance on cash over time. Second, it appears that higher rates of inflation also lower the reliance on cash.

However, it is our view that the results from the full sample obscure some important non-linear effects from inflation to payments activity. We posit that the determinants of the reliance on cash are different in low-inflation environments. To illustrate our point, we run the same regression over a low inflation sample. As a benchmark, we run the same regression over a subsample in which inflation is less than 4%:

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$^4$Robust standard errors are reported.
Table 2: Relationship Between Inflation and Reliance on Cash in Low-Inflation Sample

Compared to the regression over the entire sample, the marginal effects of inflation in reducing the importance of cash are much larger. This implies that inflation can significantly lower the need for cash in low-inflation environments. For moderate inflation rates between 4 and 6.25%, inflation does not have a statistically significant impact.

In contrast to the relationship in low-inflation environments, the relationship is much different at sufficiently high inflation rates (beyond 6.25%). In fact, there is evidence of a positive relationship. This supports arguments by Bernanke and Goodfriend that high inflation rates distort the efficiency of the payments system. The results are particularly interesting considering the small number of high inflation observations:

Table 3: Relationship Between Inflation and Reliance on Cash in High-Inflation Sample

In summary, though our analysis is not sophisticated, it does provide sufficient evidence of threshold effects from inflation to the reliance on cash. At low inflation rates, less than 4%, higher inflation promotes the efficiency of the payments system. However, at high inflation rates (greater than 6.25%), inflation distorts the payments system. Moreover, the point estimate is larger in absolute value than in low-inflation settings. As a result, high inflation is much more costly than low inflation. Thus, our results suggest it is important to consider the impact of monetary policy on the endogenous pattern of transactions in the economy. Yet, standard monetary growth models do not allow for such a mechanism.\(^5\)

3 The Model

We study an infinite horizon economy populated by a representative consumer. Time is continuous and we assume that there is no source of uncertainty in this economy. Each agent has access to a constant returns to scale technology to convert capital goods, \(k(t)\) into units of goods, \(y(t)\). The production function is such that \(y(t) = f(k(t))\), which satisfies standard Inada conditions. The capital stock depreciates at a rate \(\delta \in (0, 1)\).

\(^5\)However, Ghossoub and Reed (2005) is an exception. They posit that inflation affects the reliance on cash through the endogenous degree of production specialization. Ghossoub and Reed (2010, 2012) show that monetary policy affects the role of cash through its impact on the level of economic development. By comparison, our results show that monetary policy has a direct effect on the reliance on cash rather than indirectly through endogenous variables such as the capital stock. Moreover, we also provide empirical evidence of non-linearities in the inflation-payments relationship.
Agents derive utility from consuming the economy’s single consumption good, \( c(t) \). The lifetime utility of a typical consumer is:

\[
\int_0^\infty e^{-\rho t} u(c(t)) \, dt
\]  

(1)

where \( \rho \) is the discount rate and \( u(c(t)) \) is concave in \( c \).

As in Stockman (1981), money is introduced through a cash-in-advance constraint. In particular, agents need cash to consume and to invest in capital goods, \( k(t) \). Following previous work such as Wang and Yip (1992), a fraction \( \Gamma \in [0, 1] \) of capital investment is purchased in cash. Unlike standard cash in advance models, the reliance on cash depends on deviations from “price stability.” That is, there is an intermediate level of inflation, \( \bar{\pi} \), which minimizes the use of money for transactions. The following specification for the reliance on cash is used:

\[
\Gamma = \Gamma_0 \left[ \pi (\pi - \bar{\pi}) + a \right]
\]

where \( \pi \) is the inflation rate and \( a \) and \( \Gamma_0 \) are non-negative parameters. The scaling factor “a” serves two purposes. First, it establishes that \( \Gamma \) is non-negative when inflation is below its target. Moreover, it reflects the importance of factors other than price stability that could affect the reliance on cash. For instance, deviations of inflation from its target are of significant importance when the value of “a” is small.\(^6\) The degree of reliance on cash has the following properties:

**Corollary 1.** \( \frac{d\Gamma}{d\pi} < (\geq) 0 \) for all \( \pi < (\geq) \frac{\bar{\pi}}{2} \). \( \frac{d^2\Gamma}{d\pi^2} > 0 \) and \( \Gamma > 0 \) if \( a > \left( \frac{\bar{\pi}}{2} \right)^2 \).

Corollary 1 points out that when inflation is sufficiently below its target, higher inflation rates help improve the efficiency of the financial system by reducing the reliance on cash to finance investment activity. However, once inflation exceeds a threshold level, agents’ reliance on cash increases at an increasing rate. This assumption follows from our results that high inflation is more costly for the payments system than low inflation. In this manner, our specification of \( \Gamma \) serves to reflect how credit market conditions are exacerbated by high inflation rates.

Furthermore, each consumer faces the following cash-in-advance constraint:

\[
c(t) + \Gamma_0 \left[ \pi (\pi - \bar{\pi}) + a \right] \dot{k}(t) \leq m(t)
\]  

(2)

where \( m(t) \) is the stock of real money balances.

The consumer’s problem is to maximize (1) subject to:

\[
\dot{k}(t) + \dot{m}(t) = f(k(t)) - \delta k(t) - \pi m(t) + v(t) - c(t)
\]  

(3)

and the cash-in-advance constraint, (2), where \( v(t) \) represents the lump-sum transfer of money from the monetary authority at time \( t \).

In this manuscript we focus on the behavior of the economy in the steady-state, where \( \dot{c}(t) = \dot{m}(t) = \dot{k}(t) = 0 \). Application of Pontryagin’s Maximum Principle to solve the agent’s problem and some algebra yield the following modified golden rule equation:

\[
f'(k) = (\rho + \delta) + \rho (\rho + \pi) \Gamma_0 \left[ \pi (\pi - \bar{\pi}) + a \right]
\]  

(4)

which states that agents choose the level of capital investment up to the point where the marginal benefit of maintaining a higher steady-state stock of capital equates its cost.

\(^6\)One can assign a separate weight on the term \( \pi (\pi - \bar{\pi}) \), however the main insights remain intact.
The term, \( \rho (\rho + \pi) \Gamma_0 [\pi (\pi - \bar{\pi}) + a] \) represents that in order to acquire a higher level of capital accumulation, individuals must purchase more goods using money balances. We proceed to study the effects of inflation on capital formation.

Differentiation of (4) with respect to \( \pi \) yields:

\[
\frac{dk}{d\pi} = \left( \frac{\rho \Gamma_0}{f''(k)} \right) \left[ \pi (\pi - \bar{\pi}) + (\rho + \pi) [(\pi - \bar{\pi}) + \pi] \right]
\]  

The first component of (5) is the standard result from cash-in-advance models. For a given reliance on cash for transactions, increasing the inflation rate raises the tax on investment and reduces money holdings so that capital accumulation is affected. The second component is new to the literature— the reliance on cash versus credit for investment financing depends on the stance of monetary policy. At low levels of the inflation rate, \( \pi < \frac{\bar{\pi}}{2} \), an increase in the inflation rate helps promote “price stability” and lowers the reliance on cash.

Therefore, the model highlights two important effects of monetary policy on capital accumulation. Higher inflation reduces money holdings, but at low rates, it promotes the efficiency of the payments system and relaxes requirements for cash-dependent financing of capital. The implications of (5) are summarized in the following:

**Proposition 1.** Let \( a < \rho \bar{\pi} \). Under this condition, \( \frac{dk}{d\pi} \geq (\pi - \bar{\pi})^{\frac{3}{2}} \) if \( \pi \leq (\pi - \bar{\pi})^{\frac{3}{2}} \), where \( \pi \) solves \( \bar{\pi}^2 + \left( \frac{3}{2} \right) (\rho - \bar{\pi}) \pi + \left( \frac{3}{2} \right) (a - \rho \bar{\pi}) = 0 \).

Consistent with the empirical evidence on threshold effects from inflation to overall activity, the Proposition indicates that the effects of inflation on capital formation and output are non-monotonic. In particular, a Tobin effect is observed when inflation is sufficiently below its target, while a reverse-Tobin effect takes place when inflation is above a threshold level. The intuition is as follows. When inflation is initially low, slightly higher inflation rates encourage the use of credit to finance capital investment. This effect dominates the higher cost of capital investment due to a lower value of money that comes about from the higher inflation rate. In this manner, higher inflation rates promote capital formation and welfare when inflation is sufficiently below its target level. Once inflation exceeds the threshold level, \( \bar{\pi} \), the higher costs of inflation through lower money demand dominate the positive effects of inflation. Moreover, for inflation rates \( \pi > \bar{\pi} \), price instability distorts the payments system so that the reliance on cash rises along with inflation.

Given the asymmetry in the inflation-output relationship, one should anticipate the welfare costs of inflation to be much different than previously found in the literature. We address this issue in the following section.

### 4 Welfare Effects of Inflation

In this section, we study how the effects of inflation on credit market conditions alter the welfare costs of inflation and compare to previous work such as Cooley and Hansen (1991) and Lucas (2000). In order to do so, we parametrize the model described above and solve it numerically. Let the production function be given by: \( f(k) = k^\alpha \), where \( \alpha = 0.33 \) is the capital share of total output and following Lucas (2000), capital depreciates at a rate \( \delta = 0.025 \). Furthermore, the preferences of a representative agent are expressed by \( u(c) = \frac{c^{\beta - 1}}{1 - \beta} \), where \( 1/\beta \) is the intertemporal elasticity of substitution, with \( \beta = 2 \). As in
the growth literature, we set $\rho = 0.05$. The inflation target, $\pi$ is set at 4% to match our empirical work.\(^7\)

We proceed to discuss our choice of the parameters $a$ and $\Gamma_0$. We pick a value of “$a$” such that the result in Proposition 1 holds ($a < \rho \bar{\pi} = 0.002$). From this perspective, we choose $a = 0.001$. Finally, in order to pin down a value for $\Gamma_0$, we assume that the inflation target is set at zero, with $\Gamma = \Gamma_0 [a + \pi^2]$. By doing so, our benchmark case does not allow for any threshold effects from inflation to the efficiency of the payments system as is standard in the existing literature.

Using data for the U.S. economy, we choose $\Gamma_0$ so that an average inflation rate of 10% is associated with a 1% welfare loss. This mirrors estimates previously obtained by Cooley and Hansen (1991) and Lucas (2000). As in previous work, a consumption-based (compensating variation) measure of welfare loss is constructed. Let $c_0^*\ be the amount of consumption in a steady-state with a constant money supply and $c_1^*\ be the amount at 10% money growth. Using the functional form for preferences and the parameters discussed above, the welfare costs of inflation can simply be expressed as:

$$ \Delta W = \frac{c_0^*}{c_1^*} - 1 $$

In combination with the previous parameters, the model generates a value of $\Gamma_0 = 25.513$. This value of $\Gamma_0$ corresponds to a reliance on cash of 28% ($\Gamma(\pi) = 0.2806$) at a 10% inflation rate.

Given the set of parameters, the welfare cost of 10% inflation is 0.626%, which is significantly lower than the 1% previous found in the literature. Moreover, over the range of “$a$” that provides a non-monotonic relationship between inflation and output, the welfare costs of inflation remain significantly lower than previous estimates. As observed in Table 4 below, our model generates a maximum welfare cost of 10% inflation of 0.685%, where 20.28% of investment is financed with cash at 10% inflation. In order to reach a 1% welfare cost of inflation, a reverse-Tobin effect has to prevail, which is not consistent with recent empirical studies.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\Delta W$</th>
<th>$\Gamma(\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00100</td>
<td>0.626%</td>
<td>17.859%</td>
</tr>
<tr>
<td>0.00125</td>
<td>0.641%</td>
<td>18.497%</td>
</tr>
<tr>
<td>0.00150</td>
<td>0.657%</td>
<td>19.135%</td>
</tr>
<tr>
<td>0.00175</td>
<td>0.672%</td>
<td>19.772%</td>
</tr>
<tr>
<td>0.00195</td>
<td>0.685%</td>
<td>20.283%</td>
</tr>
</tbody>
</table>

Table 4: Welfare Cost of Inflation

5 Conclusion

The neoclassical growth model is the benchmark framework for studying the welfare effects of inflation. However, existing work likely produces misleading estimates because it does not include any mechanism for inflation to promote capital accumulation and welfare.

\(^7\)The 4% rate is fairly close to the 3.58% inflation estimate by Clarida et al. (2000) for the Volcker-Greenspan era. However, minimizing the role of cash and maximizing lifetime utility are not the same objective. The threshold $\bar{\pi}$ is less than $\pi$. 

8
Such rigid assumptions do not line up with recent evidence of a Tobin effect for advanced economies like the United States. Based upon empirical results, we present a model in which there are threshold effects from inflation to the reliance on cash. Our analysis makes considerable progress in determining the costs of inflation by acknowledging a Tobin effect occurs at low inflation rates but trying to exploit a trade-off at high rates distorts the efficiency of the payments system with strong adverse consequences for the capital stock and welfare.
References:


