Inflation Thresholds and the Efficiency of the Banking Sector

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Abstract

Empirical findings suggest that higher inflation rates stimulate economic activity when inflation is initially low. However, when inflation rises above a certain threshold level, further increases in the inflation rate hinder economic activity. Furthermore, the threshold level of inflation is often found to be much higher in developing countries compared to that in advanced economies. Interestingly, economic theory does not provide an explanation for the higher thresholds in developing countries. I present a monetary growth model where inflation has non-monotonic effects on capital formation. Moreover, I demonstrate that the threshold inflation rate is increasing with the degree of inefficiency of the banking sector. For example, the threshold rate of money growth beyond which inflation

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becomes detrimental for the real economy is much higher in economies where the banking system is less competitive.

**JEL Codes:** E31, E41, E44, O42

**Keywords:** Economic Development, Inflation Thresholds, Banking Competition, Monetary Policy

## 1 Introduction

Empirical studies find a non-linear relationship between inflation and the real economy. In particular, higher levels of inflation tend to stimulate the economy when inflation is initially low. However, once inflation crosses a certain threshold level, inflation becomes detrimental for output and growth. Furthermore, the threshold level of inflation is often found to be much higher in developing countries (around 11%) compared to that in advanced economies (around 1-3%).\(^1\)

A number of theoretical attempts have been made to explain why monetary policy can have non-linear effects. For instance, Azariadis and Smith (1996) demonstrate that inflation can exacerbate credit market frictions. In particular, higher rates of money growth (steady-state inflation) lower the real cost of borrowing which stimulates risky borrowing. In this manner, sufficiently high inflation rates induce banks to ration credit, which reduces capital accumulation.\(^2\) More recent work by Antinolfi and Kawamura (2007) examines a

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\(^2\)Choi, Smith, and Boyd (1996) construct a similar model and reach a similar conclusion.
model where two currencies can circulate in the economy. When inflation is low, agents do not hold foreign currency and all savings are intermediated. In their setting, agents receive injections from the government under a higher rate of money growth, which stimulates deposits and capital formation. However, once inflation increases beyond certain level, agents substitute foreign currency for deposits denominated in local currency, which hinders capital formation.\(^3\)

While previous work focuses on rationalizing the non-monotonic effects of inflation, there has been no formal attempts to explain why the threshold levels are much higher in developing countries. This paper provides a first attempt at addressing this issue by linking the effects of monetary policy to the degree of efficiency in the banking sector - for example the degree of banking competition. In particular, I demonstrate that the threshold inflation rate is increasing with the degree of inefficiency of the banking sector. That is, the threshold rate of money growth beyond which inflation becomes detrimental for the real economy is much higher in economies where the banking system is less competitive. Notably, the database compiled by Beck et al. (2004) indicates that the banking system in less developed countries is highly inefficient and more concentrated compared to that in advanced economies.\(^4\)

I proceed to provide a detailed description of my modeling framework. I examine a two-period overlapping generations production economy inhabited

\(^3\)A number of other studies such as Schreft and Smith (1997), Huybens and Smith (1999), Ghossoub and Reed (2010), and Ghossoub (2012) find that the effects of monetary policy vary between different steady-states. However, the relationship between inflation and the real economy is monotonic depending on which steady-state is considered.

\(^4\)See also Khemraj (2010).
by two types of agents, depositors and bankers. Following Townsend (1987),
depositors are born on one of two geographically separated, yet symmetric lo-
cations. At the end of each period and after portfolios are made, a fraction of
young depositors is randomly chosen to relocate to the other location. Due to
private information and limited communication relocated agents must liquidate
their assets (physical capital) into cash to be able to consume. As banks can
completely diversify idiosyncratic risk, all savings are intermediated. Therefore,
bankers take deposits, insure their depositors against relocation shocks, and in-
vest in the economy’s assets to maximize profits. Moreover, following Ghossoub
and Reed (2013), the deposit market is characterized by Bertrand competition,
while banks compete over quantities in the capital market. Finally, there is
a government that targets the rate of money creation (steady-state inflation).
Unlike previous work such as Schreft and Smith (1997) banks are subject to
reserve requirements.

In this setting, a change in the rate of money creation (steady-state inflation)
operates through two primary channels. First, as in the standard Tobin logic,
a higher rate of money growth raises the cost of holding money which stim-
ulates capital formation. In addition, higher inflation rates hinder the ability
of banks to provide insurance against liquidity risk, leading to fewer deposits.
When inflation is initially low, banks hold excess amount of reserves and the
Tobin effect dominates. However, once inflation exceeds a certain level, banks
can no longer economize on cash holdings due to regulatory measures. Con-
sequently, further increases in inflation hamper capital formation and output.
Interestingly, banks in economies with a less competitive banking system hold a significant amount of excess reserves, resulting in a much higher threshold effect compared to economies where the banking system is more competitive.

Notably, a number of studies document excessive liquidity holdings by banks in less developed countries. For example Saxegaard (2006) provides empirical evidence that underdevelopment in financial markets such as lack of competition in the banking sector is a primary factor explaining why banks in Sub-Saharan African countries hoard excess liquidity.\textsuperscript{5,6} Khemraj (2010) reaches a similar conclusion. More recent work by Ghossoub and Reed (2013) provide empirical support for a positive correlation between the degree of banking concentration and the level of liquidity in banks’ portfolios across OECD countries.

Finally, I study the linkages between banking competition and the level of economic activity. In particular I demonstrate that higher degrees of banking concentration are detrimental for capital formation only when financial markets are initially highly distorted and capital formation is relatively low. However, in an economy where the financial system is initially highly competitive, more concentration initially does not impact capital formation.

The result confirms recent findings of a non-linear relationship between the degree of banking concentration and economic activity. For example, in a cross-section 74 countries, Beck et al. (2004) study the impact of banking competition on firms’ access to credit. There main finding is that the availability of credit is

\textsuperscript{5}The work by Fielding and Shortland (2005) also highlights this issue.

\textsuperscript{6}Clearly, banks may hold excess reserves for many other reasons such as political instability and systemic risk. However, I focus on distortions from market power as an additional motive for banks to hold highly liquid portfolios.
lower in economies with a more concentrated banking system, especially for less
developed countries. However, the relationship turns insignificant for countries
with high GDP per capita and well developed institutions. The result by Beck
et al. (2004) was also confirmed by Deida and Fattouh (2005), who find that
banking concentration is negatively associated with per-capita income growth
and industrial growth only in low-income countries.

This paper is also related to a recent literature that examines the linkages
between monetary policy and the competitive structure of the banking system.
For example, Paal, Smith, and Wang (2005) compare the rates of growth in an
economy with a monopolistic banking system to a one with a perfectly com-
petitive system. They find that a monopoly banking system can lead to higher
growth as the bank seeks to economize on cash holding compared to when the
system is competitive. 7 Boyd et al. (2004) use a similar structure as in Paal et
al. (2005) albeit in an endowment economy and aggregate liquidity shocks. As
in Paal et al. (2005) they demonstrate that a monopolistic banking system has
an incentive to hold a less liquid portfolio, which makes it more susceptible to
a liquidity crises compared to perfect competition, especially when inflation is
low. 8 The result in both papers hinges on the fact that distortions from market
power can only arise in the deposit market. This is at odds with empirical work
such as Hannan (1991) and Corvoisier and Gropp (2002) that find a positive

\footnote{Other studies that follow a similar approach include Guzman (2000), Ghossoub et al.
(2012), and Ghossoub (2012).}

\footnote{In a sample of 69 countries Beck et al. (2006) find that banking crises are less likely
to occur in economies with more concentrated banking sectors. A similar conclusion was
reached by Arias (2010). This suggests that banks in a highly concentrated banking system
hold highly liquid portfolios.}
relationship between banking concentration and interest rates on loans.\footnote{Beck et al. (2004) examine a sample of 74 countries and find that financing obstacles are higher in economies with more concentrated banking systems. Furthermore, in a study over Latin American economies, Martinez Peria and Mody (2004) find a positive relationship between banking concentration and interest rate spreads.}

More recent work by Ghossoub and Reed (2013) studies the linkages between monetary policy and banking concentration. In their setting, there is a monotonic relationship between monetary policy and the economy. My work differs in two important manners. First, the saving choice of depositors is non-trivial in my model. This permits inflation to directly influence the level of deposits as it affects the ability of banks to provide risk sharing services. Second, the banking system is subject to legal requirements that become restrictive at high levels of inflation.

The remainder of the paper is structured as follows. In Section 2, I describe the model. The effects of monetary policy and banking competition in the steady-state are examined in Section 3. I offer concluding remarks in Section 4. Most of the technical details are presented in the Appendix.

2 Environment

Consider a discrete-time economy with two geographically separated locations or islands. Each location is populated by an infinite sequence of two-period lived overlapping generations. Let $t = 1, 2, \ldots, \infty$, index time. At the beginning of each time period, a unit mass of ex-ante identical workers and $N$ financial intermediaries (or bankers) are born on each island. Each bank is indexed by
Each young worker is endowed with one unit of labor effort which she supplies inelastically and is retired when old. Further, workers derive utility from consuming the economy’s single consumption good, $c$ in both periods of their life. The preferences of a typical worker are expressed by $u(c_1, c_2) = \frac{c_1^{1-\theta}}{1-\theta} + \frac{c_2^{1-\theta}}{1-\theta}$, where $c_1$ and $c_2$ are consumption in young and old age respectively. In addition, $\theta < 1$ is the coefficient of risk aversion and $\beta \in (0, 1)$ is an exogenous discount factor.

The consumption good is produced by a representative firm which rents capital and hires labor from young agents. The production function is denoted by $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where $K_t$ is the aggregate capital stock and $L_t$ denotes the amount of labor hired. Moreover, $A$ is an exogenous technology parameter and $\alpha \in (0, 1)$ reflects capital’s share of total output. Further, the capital stock completely depreciates in the production process.

There are two types of assets in this economy: money (fiat currency) and capital. Denote the aggregate nominal monetary base by $M_t$. At the initial date 0, the generation of old workers at each location is endowed with the aggregate capital, $K_0$ and money supply, $M_0$. Since the total population size is equal to one, these variables also represent their values per worker. Moreover, one unit of investment by a young agent in period $t$ becomes one unit of capital next period. Assuming that the price level is common across locations, I refer to $P_t$ as the number of units of currency per unit of goods at time $t$. Thus, in real terms, the supply of money per worker is, $\tilde{m}_t = M_t / P_t$.  

8
Moreover, individuals in the economy are subject to relocation shocks. Each period, after portfolios are made, agents find out that they have to relocate to the other island with probability $\pi \in (0, 1)$. Since the population size is one, $\pi$, also reflects the number of agents that must move. These agents are called “movers.” The relocation probability is publicly known and its realization is private information. Random relocation thus plays the same role that liquidity preference shocks perform in Diamond and Dybvig (1983). Limited communication and spatial separation make trade difficult between different locations.

As in standard random relocation models, fiat money is the only asset that can be carried across islands. Furthermore, currency is universally recognized and cannot be counterfeited - therefore, it is accepted in both locations.\footnote{The model can be easily modified to permit some other asset to hedge against liquidity risk as in Antinolfi and Kawamura (2007). Such change simply reinforces the primary results of this paper.}

Given the structure of the model, financial intermediaries play two primary roles in this economy. First, they pool risk by attracting enough deposits. Second, they intermediate savings by investing in the economy’s assets on behalf of their depositors.

The final agent in the economy is a central bank that follows a constant money growth rule. The aggregate nominal stock of cash in period $t$ is expressed by $M_t = \sigma M_{t-1}$, where $\sigma > 0$ is the gross rate of money creation. In real per capita terms:

$$\tilde{m}_t = \sigma \frac{P_{t-1}}{P_t} \tilde{m}_{t-1}$$

(1)
where $\frac{P_t}{P_{t-1}}$ is the real gross rate of return on money balances between period $t - 1$ and $t$. Following Huybens and Smith (1998), the government uses its seigniorage income to finance an exogenous sequence of government spending.\textsuperscript{11}

As in their work, I assume that government spending does not affect the portfolio decision of agents in the economy. Finally, the monetary authority regulates the operations of the banking sector by requiring banks to hold a minimum fraction of their deposits into cash reserves. Further, I assume that the required reserves ratio, $\rho$, is such that: $\rho \epsilon [0, \bar{\rho}]$, where $\bar{\rho} < 1$. I provide additional discussion about the upper bound on $\rho$ in the following section.

3 Trade

3.0.1 Factor markets:

Denote $r_t$ and $w_t$, to be period $t$’s rental rates for capital and labor, respectively. Perfect competition in factor markets implies that factor inputs are paid their marginal product. Using the fact that in equilibrium, $L_t = 1$, we have:

\begin{equation}
    r_t = r(K_t) = \alpha AK_t^{\alpha - 1}
\end{equation}

and

\begin{equation}
    w_t = w(K_t) = (1 - \alpha) AK_t^\alpha
\end{equation}

\textsuperscript{11}The main insights of the paper are unchanged if for example seigniorage income is rebated back to young depositors in lump-sum transfers.
3.0.2 Agents’ saving behavior

At the beginning of each period and before the shock is realized, young agents work and receive their labor income, $w_t$, which is divided between young age consumption and savings. Given that agents are subject to relocation shocks, all their savings are intermediated. Each bank announces deposit rates taking the announced rates of return of other banks as given. In particular, a bank promises a gross real return on deposits, $r^{m}_t$ if a young individual is relocated and a gross real return $r^{n}_t$ if not. Given the expected return on savings, a young agent chooses to deposit $d_t$ units of goods in the banking system.

A typical agent chooses how much to consume in each stage of her life, $c_{1,t}$ and $c_{2,t+1}$, and determines how much to save to maximize her discounted lifetime utility. An agent’s problem is summarized by:

$$\text{Max}_{c_{1,t}, c_{2,t+1}, d_t} u(c_{1,t}) + Eu(c_{2,t+1})$$

subject to young age budget constraint:

$$c_{1,t} = w_t - d_t$$

and old age consumption is:

$$c_{2,t+1} = \begin{cases} r^{m}_t d_t & \text{with probability } \pi \\ r^{n}_t d_t & \text{with probability } 1 - \pi \end{cases}$$
Moreover, using (6), the expected utility from old age consumption is:

\[ Eu(c_{t+1}) = \pi (u(r^m_t d_t)) + (1 - \pi) (u(r^n_t d_t)) \] (7)

Upon substituting the constraints into the objective function, an agent’s problem is reduced into a choice of deposits:

\[ \max_{d_t} \left( \frac{w_t - d_t}{1 - \theta} \right)^{1-\theta} + \frac{1}{1 - \theta} \left[ \pi (r^m_t d_t)^{1-\theta} + (1 - \pi) (r^n_t d_t)^{1-\theta} \right] \] (8)

It can be easily verified that the equilibrium amount of deposits is:

\[ d_t = \frac{1}{1 + \frac{1}{[\pi r^m_t + (1 - \pi) r^n_t]^{1-\theta}}} w_t \] (9)

which is strictly increasing in the return to deposits in either state of nature and the level of income. I proceed to define the saving rate, \( s_t = \frac{d_t}{w_t} \), with:

\[ s_t = \frac{1}{1 + \frac{1}{[\pi r^m_t + (1 - \pi) r^n_t]^{1-\theta}}} \] (10)

### 3.0.3 A typical bank’s problem

In this economy, banks offer similar financial services. Therefore, each bank receives the same market share in the deposit market, attracting \( d_t/N \) in deposits (or \( 1/N \) depositors). Given that banks solve the same problem, I omit the indexation for each bank. Furthermore, unlike previous work such as Schreft and Smith (1997) and Ghossoub (2012), the rental market is characterized by Cournot (quantity) competition. That is, each bank recognizes that its own
decisions about the amount of capital supplied will affect the market rental rate but that its choice does not affect that of other banks.

Each bank allocates its deposits between cash reserves and capital goods. Let \( m_t \) and \( k_{t+1} \), respectively denote the real amount of cash balances and capital goods held by each bank. By comparison to the rental market, banks compete over price in the deposit market. In this manner, a bank makes its portfolio choice to maximize the expected utility of its depositors, (8) taking as given the saving decision by agents, (10). Using (10) into (8), the bank solves the following problem:

\[
\max_{r^m_t, r^n_t, m_t, k_{t+1}} \left[ \left( 1 + \beta \left[ \pi [r^m_t]^{1-\theta} + (1 - \pi) [r^n_t]^{1-\theta} \right]^\frac{1}{\theta} \right)^\theta \frac{w_t^{1-\theta}}{1 - \theta} \right]
\]

subject to the following constraints. First, the bank’s balance sheet condition is expressed by:

\[
m_t + k_{t+1} \leq \frac{d_t}{N}; \ t \geq 0
\]

Second, as currency is the only asset that can be transported across locations, payments made to movers are made strictly out of the return on cash balances:

\[
\frac{\pi}{N} r^m_t d_t \leq m_t \frac{P_t}{P_{t+1}}
\]

In addition, I choose to study equilibria in which money is dominated in rate of return. Therefore, banks will not carry money balances between periods \( t \) and \( t + 1 \). The bank’s total payments to non-movers are therefore paid out of
its return on capital in \( t + 1 \):

\[
\frac{(1 - \pi)}{N} r_t^n d_t \leq r (K_{t+1}) k_{t+1}
\]  

(14)

As agents’ types are private information, the bank’s contract must be incentive compatible. Thus, the following self-selection constraint must hold:

\[
r_t^n \leq r_t^n
\]

(15)

Further, the bank must meet its reserve requirements, with:

\[
\gamma_t = \frac{m_t}{d_t/N} \geq \rho
\]

(16)

where \( \gamma \) is the reserves to deposits ratio. Finally, as stated above, each bank faces the market’s inverse demand for capital:

\[
r (k_{t+1}) = \alpha A (K_{t+1} (k_{t+1}))^{\alpha-1} L_{t+1}^{1-\alpha}
\]

(17)

where \( K_{t+1} = \sum_{j=1}^{N} k_j^{t+1} \).

Upon substituting the binding constraints, (12) – (14) into (11), the bank’s problem reduces to:

\[
\max_{m_t, k_{t+1}} \left( 1 + \beta \left[ \pi^\theta \left( \frac{P_t}{P_{t+1}} \right)^{1-\theta} \frac{m_t^{1-\theta}}{(m_t + k_{t+1})^{1-\theta}} + (1 - \pi)^\theta \frac{(r (K_{t+1}))^{1-\theta} k_{t+1}^{1-\theta}}{(m_t + k_{t+1})^{1-\theta}} \right]^\theta \frac{w_t^{1-\theta}}{1 - \theta} \right)
\]
As I demonstrate in the appendix, the solution to the bank’s problem generates the demand for real money balances by a typical financial intermediary:

\[ m_t = \gamma_t d_t \]

where

\[ \gamma_t = \begin{cases} \frac{1}{\frac{1}{1+\frac{1-\pi}{\pi} \frac{1}{\theta} (1-\alpha) N} \frac{\rho}{\theta}} \quad & \text{if } I_t < \hat{I} \\ \frac{\rho}{\theta} & \text{if } I_t \geq \hat{I} \end{cases} \]

\[ I_t = \frac{r_{t+1} P_{t+1}}{P_t} \] is the gross nominal return to capital between \( t \) and \( t+1 \), and

\[ \hat{I}_t = \left( \frac{1}{1-\frac{1-\alpha}{\pi} \frac{\rho}{1-\frac{\pi}{\rho}}} \right) \frac{\theta}{\pi}. \]

As in standard monetary models, banks demand less money if its return falls relative to other assets. Moreover, banks hold excess reserves when the opportunity cost of holding money is very low. As the return to capital exceeds its threshold level, \( \hat{I} \), the reserve requirements constraint binds and banks can no longer economize on cash holdings due to financial regulation. Interestingly, the threshold nominal return to capital is much higher when the banking system is more concentrated. It can be clearly seen from (19) that for a given nominal return to capital, banks hold more excess reserves when the banking system is less competitive. Therefore, it requires a much higher nominal return to capital to exhaust all the excess reserves compared to an economy with a more competitive banking sector.

Upon substituting the expressions for money demand, into the bank’s bal-
ance sheet, the supply of capital by a bank in $t + 1$ is given by:

$$k_{t+1} = \begin{cases} 
(1 - \gamma (I_t)) d_t / N & \text{if } I_t < \hat{I} \\
(1 - \rho) d_t / N & \text{if } I_t \geq \hat{I}
\end{cases} \quad (20)$$

Finally, using (13), (14), and (19), the relative return to depositors can be expressed by:

$$r^n_t / r^m_t = \begin{cases} 
\left(1 - \left(\frac{1 - \alpha}{N}\right)\right)^{\frac{1}{\pi}} I_t^{\frac{1}{\gamma}} & \text{if } I_t < \hat{I} \\
\left(1 - \frac{1 - \rho}{\rho}\right)^{\frac{1}{\pi}} I_t & \text{if } I_t \geq \hat{I}
\end{cases} \quad (21)$$

which indicates that depositors receive a lower amount of insurance against relocation shocks when the return to capital is high relative to that on cash balances.

In addition, the incentive compatibility constraint holds if $I_t \geq \frac{1}{1 - \left(\frac{1 - \alpha}{N}\right)} \equiv L$ with $L > 1$. Throughout the analysis, I will assume that: $\hat{I} > L$, which permits banks to hold excess reserves in equilibrium. This condition can be written as a one on $\rho$:

$$\rho < \frac{1}{1 + \frac{1 - \pi}{\pi} \left[1 - \frac{1 - \alpha}{N}\right]} = \tilde{\rho} < 1 \quad (22)$$

As shown in Bhattacharya et al. (1997), albeit in a competitive environment, agents will not intermediate their savings if the financial system is highly repressed ($\rho > \tilde{\rho}$). Therefore, we assume that the condition in (22) always holds.
3.1 General Equilibrium

I now combine the results of the preceding section and characterize the equilibrium for the economy. In equilibrium labor receives its marginal product, (3), and the labor market clears:

\[ L_t = 1 \]  

(23)

Using the equilibrium expressions for \( r_m^m \) and \( r_n^m \) into the savings of young agents to get the general equilibrium level of the saving rate:

\[
s_t \equiv s(I_t) = \begin{cases} 
1 + \frac{\gamma_t[1 - \frac{1-\omega}{\alpha} \gamma_t \pi]}{\beta_t[1 - \frac{1-\omega}{\alpha} \gamma_t]} & \text{if } I_t < \hat{I} \\
1 + \frac{\left(\frac{r_{t+1}}{r_t}\right)^{\frac{1-\eta}{\eta}}}{\beta_t \left[\pi \left(\frac{1-\eta}{1-\gamma_t}\right)^{1-\eta} + (1-\gamma_t)I_t^{1-\eta}\right]^{\frac{1-\eta}{\eta}}} & \text{if } I_t \geq \hat{I} 
\end{cases}^{-1}
\]

(24)

I proceed with the following observations:

**Lemma 1.** \( s'(I_t) > 0 \), \( s(I) = \left[1 + \frac{1}{\beta_t \left(\frac{\gamma_t(I_t)}{1-\gamma_t}\right)^{1-\eta}}\right]^{-1} \), \( \lim_{I_t \to \infty} s \to 1 \), and \( \frac{\partial s_t}{\partial r_n^m} < 0 \).

In words, agents save more when the return to capital is higher. In the limit, agents do not value young age consumption when the return on capital is infinitely large. Analogously, agents save less when inflation increases as it erodes the value of money and the expected return on savings for a given return
to capital.\footnote{Bayoumi and Gagnon (1996) provide empirical support for a negative relationship between inflation and the saving rate in OECD countries.}

Using (1) and (18), the money market is in equilibrium when:

\begin{equation}
\gamma (I_{t+1}) s_{t+1} w (K_{t+1}) = \frac{P_{t+1}}{P_t} \gamma (I_t) s_t w (K_t) \quad \text{if } I_t < \hat{I} \\
\gamma (I_{t+1}) s_{t+1} w (K_{t+1}) = \frac{P_{t+1}}{P_t} \gamma (I_t) s_t w (K_t) \quad \text{if } I_t \geq \hat{I}
\end{equation}

Furthermore, using (2), (9), (10), and (20), the aggregate supply of capital by banks is such that:

\begin{equation}
K_{t+1} = \begin{cases} 
(1 - \gamma (I_t)) s (I_t) w (K_t) & \text{if } I_t < \hat{I} \\
(1 - \rho) s (I_t) w (K_t) & \text{if } I_t \geq \hat{I}
\end{cases}
\end{equation}

The capital market clears, when the supply of capital by banks, (26), is equal to its demand by firms:

\begin{equation}
I_t = \frac{P_{t+1}}{P_t} \alpha AK_{t+1}^\alpha - 1
\end{equation}

Conditions (24) – (27), characterize the behavior of the economy at each point in time. In this paper, I focus on stationary equilibria. Thus, I turn to study the behavior of the economy in the steady-state.

### 3.2 Steady-State Analysis

Imposing steady-state on the money market clearing condition, (25) implies that the rate of money creation, $\sigma$, is equal to the rate of inflation, $\frac{P_{t+1}}{P_t}$ in the long
run. The steady-state behavior of the economy is summarized by equilibrium in the capital market. In particular, from the bank’s balance sheet and the savings’ function of young depositors, the supply of capital by banks is:

\[
\Omega(K) = \begin{cases} 
(1 - \gamma(I)) s(I) & \text{if } I_t < \hat{I} \\
(1 - \rho) s(I) & \text{if } I_t \geq \hat{I} 
\end{cases}
\] (28)

where \(\Omega(K)\) is the aggregate capital to wage ratio \((K/w(K))\) and \(s(I)\) is given by (24).

Moreover, by imposing steady-state on (27), the inverse demand for capital by firms can be expressed by:

\[
I = \sigma\alpha AK^{\alpha - 1}
\] (29)

The steady-state behavior of the economy is characterized by the supply and demand for capital in the steady state, (28) and (29), respectively.

It is clear from (29), that the demand for capital is strictly decreasing in \(I\) as shown in Figure 1 below. In addition, Lemma 2 describes the behavior of the supply of capital, (28), in the steady-state:

**Lemma 2.** The locus defined by (28) satisfies the following: \(\frac{dK}{dt} > 0\), \(\lim_{I \to \infty} \Omega(K) \to (1 - \rho)\), and \((L, K)\) is a point on (28), where \(K: \Omega(K) = (1 - \gamma(L)) s(L)\).

In this setting, a change in the nominal return to capital has two effects on the amount of capital supplied by banks. First, a higher return to capital raises
the expected return on savings, which encourages agents to reduce their young age consumption and increase their savings. The higher amount of deposits raises banks’ ability to invest in the economy’s assets. Additionally, a higher nominal return to capital raises the cost of holding money, which encourages banks to allocate a larger fraction of their deposits into physical capital.

When the cost of holding money is sufficiently high \((I \geq \hat{I})\), banks can no longer economize on cash holding as the reserves constraint binds. Consequently, banks allocate a constant fraction of their deposits into capital for all \(I \geq \hat{I}\). However, banks receive higher deposits when the return to capital is higher, which enables them to supply more capital. An illustration of (28) and (29) is presented in Figure 1 below.

![Figure 1. Steady-State Equilibrium](image-url)
In the following Proposition, I establish existence and uniqueness of stationary equilibria. For this purpose I designate a steady-state equilibrium by a superscript, $\ast$.

**Proposition 1.** Suppose $\sigma \geq \sigma$, where $\sigma : I^* = I$. Under this condition, a banking equilibrium exists and is unique.

By Lemma 2 and the description of (29), the capital market always clears at some point like $E$ in Figure 1. However, in equilibrium the incentive compatibility constraint, (15) must hold and money and capital must both be held. From (21), both conditions are satisfied when the nominal return to capital is above $L$. This takes place if we have an excess demand for capital at $L$. Upon imposing $I = L$ on the capital market, an excess demand occurs when the rate of money creation is sufficiently large, $\sigma > \sigma$. For all $\sigma \geq \sigma$, $I \geq L$, and an equilibrium exists and is unique.

I proceed to study the effects of monetary policy.

**Proposition 2.** Define $\hat{\sigma} : I^* = \hat{I}$, with $\hat{\sigma} > \sigma$, $\frac{dK}{d\sigma} > (\leq) 0$ if $\sigma < (>) \hat{\sigma}$. In addition $\frac{dI}{d\sigma} > 0$ for all $\sigma > \sigma$.

In this setting, a change in the rate of money creation affects the economy through two primary channels. First, as in the standard Tobin logic, a higher rate of money growth raises the cost of holding money which stimulates capital formation (asset substitution). However, higher inflation rates hinder the ability of banks to provide insurance against liquidity risk, leading to fewer
deposits. When inflation is initially low, banks hold excess reserves and the asset substitution channel dominates. However, once inflation exceeds a certain level, banks can no longer economize on cash holdings due to regulatory measures. Consequently, further increases in inflation hamper capital formation and output.\textsuperscript{13}

Interestingly, the threshold level of inflation beyond which inflation has adverse effects on the economy varies with the degree of banking competition. In particular, as I demonstrate in the appendix, the threshold rate of money growth decreases with the extent of banking competition. Intuitively, when the banking system is not competitive, financial markets are highly distorted as banks hoard cash reserves and allocate few resources towards capital formation. Therefore, it takes a much higher inflation rate to drain all the excess reserves in the system, beyond which inflation has adverse effects on capital formation.

The analysis suggests that in an economy where the banking system is highly competitive, such as the United States, banks do not hold a significant amount of excess reserves, which has been the case until the recent financial crisis. Therefore, a small amount of inflation may be sufficient to hinder the functioning of the financial system and have adverse effects on capital formation. The opposite holds in economies where the banking system is not too competitive.

The effects of banking competition are examined in the following Proposition:

\textsuperscript{13}The effects of monetary policy when the reserves constraint binds are analogous to those in Haslag (1998) who examines a perfectly competitive environment with endogenous growth.
Proposition 3. Suppose $N < \hat{N}$, where $\hat{N} : I^* = \hat{I}$. Under this condition:

$$\frac{dK}{dN} > 0, \frac{dI}{dN} > 0, \text{ and } \frac{d\gamma}{dN} < 0.$$  

By comparison, if $N \geq \hat{N}$, $\frac{dK}{dN} = \frac{dI}{dN} = \frac{d\gamma}{dN} = 0$.

The intuition behind Proposition 3 is as follows. When the banking system becomes more competitive, banks economize on cash holding and supply more capital. The higher supply of capital lowers its return. However, when the degree of banking competition exceeds a certain level, banks can no longer economize on cash holding as they need to satisfy their reserve requirements. As a result, the aggregate supply of capital does not change with the degree of banking competition.

4 Conclusion

Empirical evidence points out to a non-monotonic relationship between inflation and the real economy. In particular, higher inflation rates become detrimental for the real economy when inflation exceeds a certain threshold level. Furthermore, the threshold inflation rates are much higher for developing countries. In this paper, I develop a general equilibrium model that is capable of addressing such threshold effects observed in the literature. More importantly, I demonstrate that the threshold level of inflation is much higher when the financial sector is highly distorted and not competitive. Notably, lack of competition and inefficiency are two main characteristics of the financial system in developing countries.
References


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Technical Appendix

1. Proof of Proposition 1. From the expression for the supply of capital, (28):

\[ \Omega(K) = (1 - \gamma(I)) s(I) \]

At the lower bound on \( I \), \( I_L \) it is easy to show that:

\[ \gamma(I) = \frac{1}{1 + \frac{1 - \pi}{\pi} \left[ 1 - \frac{1 - \alpha}{N} \right]} \]

Moreover, from (10) and simple algebra:

\[ s = \frac{1}{1 + \frac{1}{\beta \left[ \frac{1}{\sigma \pi} \right]^\frac{1 - \theta}{\theta}} \left( \frac{1 - \theta}{\theta} \right)^\frac{1 - \theta}{\theta} \frac{1 - \alpha}{N}} \]

where at \( I_L, \frac{r^n}{\pi} = 1 \). Moreover, from (13) and (21):

\[ r^m_i = \gamma \left[ 1 - \frac{1 - \alpha}{N} \right] \frac{1}{\sigma \pi} I^z \]

Therefore:

\[ r^n(I) = \frac{\gamma(I)}{\sigma \pi} \]

and
Furthermore, under a Cobb-Douglas production function and the definition of $I$:

$$s(I) = \frac{1}{1 + \frac{1}{\beta \left( \frac{\gamma(I)}{\sigma(I)} \right)^{1 - \theta}}}$$

Existence requires:

$$\frac{\alpha \sigma}{(1 - \alpha) I} \geq (1 - \gamma(I)) s(I)$$

Upon substituting for $L, \gamma(I),$ and $s(I)$ the condition becomes:

$$\left( \frac{\pi}{1 - \pi} + \left[ 1 - \frac{1 - \alpha}{N} \right] \right) \frac{\alpha}{(1 - \alpha)} \geq \frac{s(I)}{\sigma}$$

where $\frac{s(I)}{\sigma}$ is strictly falling in $\sigma$. Therefore, there exist a $\sigma$ such that the above holds with equality and for all $\sigma \geq \sigma, I^* \geq L$. This completes the proof of Proposition 1.

2. **Proof of Proposition 2.** I begin by showing that inflation stimulates capital formation when $I < \dot{I}$. From (28) and (24), the supply of capital is:

$$\Omega(K) = \frac{(1 - \gamma(I))}{1 + \frac{\gamma(I)(\frac{1 - \gamma(I)}{\sigma(I)} \frac{1 - \alpha}{\gamma(I)}}{\beta \left[ 1 - \frac{1 - \alpha}{\gamma(I)} \right]^\theta (1 - \sigma \left( \frac{1 - \gamma(I)}{\sigma(I)} \right)^{1 - \theta}} \right)}$$
Upon using the definition of $\gamma$, (19) and some simplification, (30) can be written as:

$$\Omega(K) = \frac{1}{(1-\gamma(I))} + \frac{1}{(1-\pi) \frac{1}{\beta} \frac{1}{\gamma(I)} \frac{1}{\beta} \frac{1}{N \gamma(I)}}$$

Finally, using the demand for capital, (27), the definition of $I = r \sigma$, and Cobb-Douglas, into (31), the following polynomial yields a solution for $r$:

$$\frac{1}{(1-\gamma(I))} + \frac{1}{(1-\pi) \frac{1}{\beta} \frac{1}{\gamma(I)} \frac{1}{\beta} \frac{1}{N \gamma(I)}} = \frac{1}{\alpha}$$

Define $\Psi(r, \sigma) \equiv \frac{1}{(1-\gamma(I))} + \frac{1}{(1-\pi) \frac{1}{\beta} \frac{1}{\gamma(I)} \frac{1}{\beta} \frac{1}{N \gamma(I)}}$. Differentiating the polynomial with respect to $\sigma$ yields:

$$\frac{dr}{d\sigma} = \frac{\partial \Psi}{\partial r} - \frac{\partial \Psi}{\partial r} < 0$$

since it is trivial to show that $\frac{\partial \Psi}{\partial r} < 0$ and $\frac{\partial \Psi}{\partial \sigma} < 0$. In this manner, the real return to capital is lower (capital stock is higher) when inflation increases.

I proceed to demonstrate that a reverse Tobin effect is present if $I \geq \hat{I}$.

Under this range of the return to capital, the equilibrium condition is:

$$\Omega(K) = (1-\rho) \frac{1}{1 + \frac{\beta}{\gamma(I)} \beta \frac{1}{\gamma(I)} \frac{1}{\beta} \frac{1}{N \gamma(I)}}$$

Upon using the demand for capital, (27), the definition of $I = r \sigma$, and Cobb-Douglas, into (33), the following polynomial yields a solution for $r$ when $I \geq \hat{I}$:
\[ \Gamma(r, \sigma) = (1 - \rho) \frac{1 - \alpha}{\alpha} r \]  \hspace{1cm} (34)

where \( \Gamma(r, \sigma) \equiv 1 + \frac{1}{\beta^\rho \left[ \frac{1}{\rho} - \frac{1}{1 - \rho} \right]^{\frac{1}{\rho}} + (1 - \pi)^{1 - \rho}} \) (1 - \rho) \frac{1 - \alpha}{\alpha} < 0.\)

Differentiation of (34) with respect to \( \sigma \) yields:

\[ \frac{dr}{d\sigma} = \frac{\partial \Gamma}{\partial \sigma} \left[ (1 - \rho) \frac{1 - \alpha}{\alpha} - \frac{\partial \Gamma}{\partial r} \right] > 0 \]

since \( \frac{\partial \Gamma}{\partial \sigma} > 0 \), while \( \frac{\partial \Gamma}{\partial r} < 0 \).

The final step is to show that the nominal return to capital increases across all the \( I \) domain. Upon using the definition of \( I \), the polynomial, (32) can be written as:

\[ \Psi_1(I, \sigma) = 1 - \frac{\alpha}{\alpha} I \]  \hspace{1cm} (35)

where \( \Psi_1(I, \sigma) \equiv \frac{\sigma}{(1 - \gamma(I))} + \frac{\sigma^\beta}{(1 - \pi) \frac{1 - \alpha}{\alpha} \beta^\rho [1 - \frac{1 - \alpha}{\alpha} \gamma(I)]^\rho} \). Differentiating with respect to \( \sigma \):

\[ \frac{dI}{d\sigma} = \frac{\partial \Psi_1}{\partial \sigma} \left( (1 - \frac{1 - \alpha}{\alpha} - \frac{\partial \Psi_1}{\partial I} \right) > 0 \]

since \( \frac{\partial \Psi_1}{\partial \sigma} > 0 \) and \( \frac{\partial \Psi_1}{\partial I} < 0 \). Analogously, it is trivial to show that (34) can be written as:

\[ \Gamma_1(I, \sigma) = (1 - \rho) \frac{1 - \alpha}{\alpha} I \]  \hspace{1cm} (36)
\[ \Gamma_1 (I, \sigma) \equiv \sigma + \frac{\sigma^\frac{1}{\rho} \left[ \prod_{\rho} \left( \frac{1 - \rho}{1 - \alpha} \right) \right]^\frac{1}{\alpha} \left[ \prod_{\rho} \left( \frac{1 - \rho}{1 - \alpha} \right) \right]^\frac{1}{\alpha}}{\beta} \]  

with:

\[
\frac{dI}{d\sigma} = \frac{\partial \Gamma_1}{\partial \sigma} - \frac{\partial \Gamma_1}{\partial I} > 0
\]

given that \( \frac{\partial \Gamma_1}{\partial \sigma} > 0 \) and \( \frac{\partial \Gamma_1}{\partial I} < 0 \). In this manner for all \( I \geq \hat{I} \), \( \frac{dI}{d\sigma} > 0 \).

The final step is to identify the threshold rate of money creation, \( \hat{\sigma} \), where \( \hat{\sigma} : I^* = \hat{I} \), with \( \gamma \left( \hat{I} \right) = \rho \). Upon using the definition of \( \hat{I} \) into the polynomial in \( I \), (36), \( I^* = \hat{I} \) if:

\[
\left[ \sigma + \frac{\sigma^\frac{1}{\rho} \left[ \prod_{\rho} \left( \frac{1 - \rho}{1 - \alpha} \right) \right]^\frac{1}{\alpha} \left[ \prod_{\rho} \left( \frac{1 - \rho}{1 - \alpha} \right) \right]^\frac{1}{\alpha}}{\beta} \right] \left[ 1 - \frac{1 - \alpha}{N} \right]^{\frac{1}{\alpha}} = \frac{(1 - \rho)(1 - \alpha)}{\alpha} \left( \frac{\gamma}{1 - \pi} \right) \frac{\rho}{\alpha}
\]

which has a unique solution, \( \hat{\sigma} \). For all \( \sigma \geq \hat{\sigma}, I \geq \hat{I} \). Finally, it is trivial to show that \( \frac{d\hat{\sigma}}{dN} < 0 \). This completes the proof of Proposition 2.

Note also that there exists an \( N, \hat{N} \), beyond which, \( I > \hat{I} \). This suggests that banks hold more cash reserves at lower values of \( N \). Equivalently, the constraint binds in a highly competitive environment. Therefore, \( \frac{dK}{dN} > 0 \) if \( N < \hat{N} \) and \( \frac{dK}{dN} = 0 \) if \( N \geq \hat{N} \).