Is the stationarity of the yen real exchange rates a puzzle?

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Abstract

This paper attempts to shed light on the puzzling finding that the results of applying the conventional or nonlinear unit root tests to the yen real exchange rates (RERs) in some recent studies appear to be rather sensitive to whether or not including the data of recent decade in the studies. It is found that this sensitivity of the test results may come from the failure to take into account the large rise and fall in the yen RERs. Using the newly developed unit root tests which account for the presence of multiple smooth temporary breaks in the RERs, the results clearly show that the yen RERs in the post-Bretton Woods period can be characterized as being linear or nonlinear stationary around infrequent smooth temporary mean changes, supporting the validity of PPP.

Keywords: Yen real exchange rates; Nonlinear stationarity; Large swings; PPP; Unit root tests

JEL classification: F31; G15

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1. Introduction

Purchasing Power Parity (PPP) is a major building block of many economic theories. PPP assumes that real exchange rate reverts to a constant mean or its equilibrium value over time. The validity of PPP can be examined by testing whether the real exchange rate (RER) has a unit root. If the unit root null hypothesis can be rejected in favor of a stationary alternative, then there is mean reversion in RER and, hence, PPP is validated. On the other hand, if the real exchange rate follows a random walk without reverting back to the equilibrium value over time, then PPP does not hold.

Empirical studies based on linear models failed to confirm the validity of PPP.1 Recent studies have therefore considered the possibility of nonlinear mean reversion in real exchange rates. As explained in Taylor (2003) and Sarno and Taylor (2001), the sources of nonlinearity in RERs may come from transaction costs, the effects of official intervention, and the interaction of heterogeneous agents in the foreign exchange market. Studies (e.g., Taylor et al., 2001; Sarno and Taylor, 2002; Sarno et al., 2004; Alba and Park, 2005; Bahmani-Oskooee et. al., 2007 and 2008) using the tests that account for non-linearity in time series movement tend to reveal the mean-reverting properties of RERs for more countries, i.e., more supportive evidence for PPP.

In a recent study, Christopoulos and León-Ledesma (2010) argue that “although the evidence regarding the recent historical period of floating exchange rates since the break-down of the Bretton-Woods system increasingly supports mean reversion, this evidence is still not as conclusive” (p. 1077). This observation holds particularly for the case of the yen real exchange rate. For example, in an analysis of the relationship between the Japanese yen and the currencies of other Asian economies, Aggarwal et al. (2000) point out that “If we can find evidence in

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1 For a recent survey of these studies and others, see Sarno (2005) and Bahmani-Oskooee and Hegerty (2009).
favour of the PPP in these real Japanese yen exchange rates, it would be strong evidence of the integration between the Japanese economy and those of other Asian countries. However, many of the empirical works devoted to the PPP hypothesis have often refuted this” (p. 351). In addition, Cheung and Lai (2001) state that “researchers have been confronted with comparable, if not greater, difficulty in detecting PPP reversion in real yen rates as opposed to real dollar rates” (p. 116).

While earlier studies reported in Cheung and Lai (2001) fail to support PPP with the yen RERs, conflicting evidence on the stationarity of the yen RERs has also been found in a number of recent studies. For example, employing the tests developed by Kapetanios, Shin, and Snell (KSS hereafter) (2003) to examine the stationarity of the yen RERs for a sample of period 1960-2000, as well as for a sub-sample period 1974-2000, Chortareas and Kapetanios (2004) found that “the yen real exchange rate may be stationary after all” (p. 113). However, it is revealed in Zhou, Bahmani-Oskooee and Kutan (hereafter, ZBK) (2008) that the test results for the stationarity of the yen RERs are different for the sample periods with or without the inclusion of the data of the 2000s. They obtained some evidence for the yen-dollar, yen-French currency, and yen-German currency RERs to be stationary when applying the KSS tests to a sample of period 1973-1998, yet no such evidence is found for a sample of period 1973-2006. Such inconsistency in the test results is also reflected in the studies of Liew et al. (2004) and Zhou (2008). With the use of the conventional and KSS unit root tests, while Liew et al. (2004) conclude that 60% of the yen RERs are stationary for a sample of period 1968-2001, Zhou (2008) claims a failure to reject the null of nonstationarity for most of the yen RERs with the data of 1968-2005. A puzzle thus arises from these recent studies: why the results of the unit root tests are so sensitive to whether or not including the data of recent decade in the studies?
The current study attempts to solve the puzzle regarding the sensitivity of the test results for the stationarity of the yen RERs to different sample periods. Given the importance of Japan in global trade flows, testing the stationarity of yen RERs is significant for reasons beyond the basic PPP theory. If the yen RERs are stationary, it means that the effects of a shock to the real exchange rates would be only temporary over time, suggesting that yen-based real exchange shocks would not have long-term detrimental effects on nations’ competitiveness and hence trade flows. This issue has significant policy implications.

The study is inspired by the recent research of Christopoulos and León-Ledesma (2010) who propose that RERs could be mean-reverting around an infrequent smooth-breaking mean that is compatible with PPP. It can be seen in the data of the bilateral yen RERs for the post-Bretton Woods period, there is a trend of large real appreciations of the yen in the 1980s and 1990s followed smoothly by large real depreciations of the yen in the recent decade. We thus point out that the sensitivity of the test results to different sample periods in those recent studies may come from their failure to take into account the large rise and fall in the yen RERs. Because the KSS tests for de-trended series allow for a linear trend in the data, such a trend may approximately present the large real appreciations of the yen in the 1980s and 1990s. Correspondingly, the KSS statistics for the samples without the inclusion of the data of recent years may show that the yen RERs are (nonlinear) stationary around a linear trend. However, such tests would not be able to account for both large real appreciations in earlier years and large real depreciations of the yen in the recent decade, and thus would fail to reject the null of a unit root in the yen RERs for the samples including the data of recent years.

In Christopoulos and León-Ledesma (2010), new tests have developed that not only allow nonlinearity in the RERs but also account for the presence of multiple smooth temporary
breaks in the RERs. They have illustrated that the results of their tests reject the null of a unit root for the majority of the dollar RERs in their study. We apply the tests of Christopoulos and León-Ledesma (2010) to the yen RERs of a group of industrial and newly industrialized countries for the post-Bretton Woods period to check (a) if their models can well capture the large swings in the yen RERs and (b) whether or not the yen RERs are stationary around large temporary mean changes.

The paper proceeds as follows. The next section describes the methodology and test procedures employed in the study. The empirical test results are presented in Section 3. The last section concludes.

2. Methodology and test procedures

The unit root tests of Kapetanios, Shin, and Snell (2003) and the tests newly developed by Christopoulos and León-Ledesma (2010) are utilized to examine the stationarity of the yen real exchange rates. The two tests have the same null hypothesis of a unit root. While both tests allow for nonlinear stationarity in the alternative hypothesis, the tests of Christopoulos and León-Ledesma (2010) also account for the presence of multiple smooth temporary breaks such as large swings in the RERs. Hence if the KSS tests are unable to reject the null of a unit root in RERs, while the Christopoulos and León-Ledesma tests reject the null hypothesis, we can then conclude that the difference in inferences is due to the presence of multiple smooth temporary breaks in RERs and PPP holds.

For \( y_t \) being the series of interest and \( u_t \) being the de-meaned or de-meaned and detrended version of \( y_t \), the KSS tests are based on the following auxiliary regression:
\[ \Delta u_t = \delta u_{t-1}^3 + \sum_{j=1}^{\rho} \rho_j \Delta u_{t-j} + \text{error} \] (1)

which is obtained from a first-order Taylor series approximation of an exponential smooth transition autoregressive (ESTAR) model specified in KSS (2003). The null hypothesis of nonstationarity to be tested with (1) is \( H_0: \delta = 0 \) against the alternative of (nonlinear) stationarity \( H_1: \delta < 0 \). The augmentations \( \sum_{j=1}^{\rho} \rho_j \Delta u_{t-j} \) are included to correct for serially correlated errors.\(^2\)

KSS (2003) use the \( t \)-statistic for \( \delta = 0 \) against \( \delta < 0 \) and tabulated the asymptotic critical values of the test statistics via stochastic simulations.

The tests developed by Christopoulos and León-Ledesma (2010) use trigonometric variables to capture large changes in the mean of \( y_t \) with the consideration of the following model:

\[ y_t = \lambda_0 + \lambda_1 \sin \left( \frac{2\pi kt}{T} \right) + \lambda_2 \cos \left( \frac{2\pi kt}{T} \right) + \nu_t \] (2)

where \( \lambda_0 + \lambda_1 \sin \left( \frac{2\pi kt}{T} \right) + \lambda_2 \cos \left( \frac{2\pi kt}{T} \right) \) is a form of Fourier function that may capture several smooth breaks of unknown form in \( y_t \). In equation (2), \( k \) is the number of frequencies of the Fourier function, \( t \) is a trend term, \( T \) is the sample size, and \( \pi = 3.1416 \). The null hypothesis with (2) is that there is a unit root in \( \nu_t \) and the alternative is that \( \nu_t \) is linear or nonlinear stationary.

The empirical investigation is conducted first by applying the conventional unit root tests, the augmented Dickey-Fuller (ADF) tests, and KSS tests to a set of yen RERs using the sample

\(^2\) See Kapetanios et al. (2003) for more details.
of the post-Bretton Woods period. To see how sensitive are the test results to the extension of the sample periods to recent years, the tests are carried out for the sample period utilized in Chortareas and Kapetanios (2004), i.e., 1974-2000, and then for the sample extended to the ending period of ZBK (2008), i.e., 1974-2006, and to the current year, i.e., 1974-2010. The test statistics obtain from regression (1) are referred to KSS1 for de-meaned data, and KSS2 for de-meaned and de-trended data. The ADF test statistics are denoted as ADF1 for the model with a constant only, and ADF2 for the model with a constant and a time trend. The number of augmentations $p$ for either the ADF tests or the KSS tests is selected based on significance testing procedure in Ng and Perron (1995). The maximum number of $p$ was set to 4 for our quarterly data, and insignificant augmentation terms were excluded.\footnote{It is found that the tests with a fixed number of augmentations, $p = 4$, or with selected number of augmentations yield very similar results. In other words, the results of the tests are not very sensitive to the models with a few more insignificant augmentation terms. To save space, only the results with selected number of augmentations are reported. The rest of the results are available from the authors upon request.}

As the results of the above tests show that the test statistics are rather different for the sample periods with or without the inclusion of recent years, the tests of Christopoulos and León-Ledesma (2010) are conducted through a three step procedure proposed in their article. Step 1 is to run an OLS regression of the yen RER ($y_t$) on a constant, $\sin \left(\frac{2\pi kt}{T}\right)$, and $\cos \left(\frac{2\pi kt}{T}\right)$ for values of $k$ between 1 and 5 and select $\tilde{k}$ that minimizes the residual sum of squares. The OLS residual series $\hat{v}_t$ is obtained for the next step. That is,

$$\hat{v}_t = y_t - \left[ \hat{\lambda}_0 + \hat{\lambda}_1 \sin \left(\frac{2\pi \tilde{k}t}{T}\right) + \hat{\lambda}_2 \cos \left(\frac{2\pi \tilde{k}t}{T}\right) \right]$$

(3)

The second step is to test for a unit root in $\hat{v}_t$ with the following regressions:
\[ \Delta \hat{\nu}_t = \alpha \hat{\nu}_{t-1} + \sum_{j=1}^{p} \rho_j \Delta \hat{\nu}_{t-j} + \varepsilon_t \]  
\[ \Delta \hat{\nu}_t = \delta \hat{\nu}_{t-1}^3 + \sum_{j=1}^{p} \rho_j \Delta \hat{\nu}_{t-j} + \varepsilon_t \]  
(4)  
(5)

where (4) is called Fourier-ADF (FADF) test with the alternative hypothesis of \( \hat{\nu}_t \) being linear stationary. (5) corresponds to regression (1) and thus is considered as a Fourier-KSS (FKSS) test with the alternative that allows \( \hat{\nu}_t \) to be nonlinear stationary. The critical values of the \( t \)-statistic for the null of \( \alpha = 0 \) or \( \delta = 0 \) against the alternative of \( \alpha < 0 \) or \( \delta < 0 \), respectively, are tabulated in Christopoulos and León-Ledesma (2010) with different sample sizes via the Monte Carlo simulations.

The last step is an F-test, \( F(\tilde{k}) \), for the significance of \( \lambda_1 \) and \( \lambda_2 \) in equation (2) if the null of a unit root in \( \hat{\nu}_t \) in step 2 is rejected. The null hypothesis is linearity, \( H_0: \lambda_1 = \lambda_2 = 0 \), against the alternative of nonlinearity, \( H_1: \lambda_1 \) and/or \( \lambda_2 \neq 0 \). A rejection of the null would indicate that the yen RER is stationary around some large changes in the mean of the RER. The critical values of the \( F(\tilde{k}) \) statistic are taken from Table 1 of Becker et al. (2006).

3. Data and empirical test results

Quarterly consumer price indices and end-of-period bilateral nominal exchange rates are obtained from the International Monetary Fund (IMF)’s International Financial Statistics online. The sample period runs from the first quarter of 1974 to the second quarter of 2010. Because the maximum number of lag length in equations (1), (4) and (5) was set to be 4, the first 5 quarterly
observations are used to compute the lagged RER changes for the tests. Therefore, the sample period effectively starts from the second quarter of 1975.

Bilateral yen real exchange rates are computed for those of the other G7 countries (i.e., Canada, France, Germany, Italy, the U.K., and the U.S.), two Pacific Basin industrial countries (i.e., Australia and New Zealand) and two Asian newly industrialized countries, Korea and Singapore. For $s_j$ and $s_i$ being the logs of Japanese and country $i$’s currency prices of a US dollar respectively, and $p_j$ and $p_i$ being the logs of the price indices of Japan and country $i$ respectively, the yen RER against country $i$’s currency is constructed as $rer_i = s_i - p_i - s_j + p_j$. Thus, a rise in $rer_i$ implies a real yen appreciation against country $i$’s currency. For 1999-2010, the dollar exchange rates of France, Germany, Italy are calculated by $s_i = s_{euro} + s_q$, where $s_{euro}$ is the log of the euro price of a dollar and $s_q$ is the log of a euro-zone country’s currency conversion rate of a euro (irrevocably fixed at the rates set on January 1, 1999).

Table 1 reports the ADF and KSS test statistics of the yen RERs for the sample periods of 1974-2000, 1974-2006, and 1974-2010. The results for 1974-2000 are very similar to those in Chortareas and Kapetanios (2004). The KSS2 statistics for de-meaned and detrended data reject the null of nonstationarity for all cases except Singapore, and thus seem to suggest that most of the yen RERs are stationary around a linear time trend. However, the results are very different when the sample periods are extended to recent years. The null of a unit root can be rejected for only 4 out of 10 cases with the sample period 1974-2006, and for 6 of 10 yen RERs with a sample of 1974-2010. Such sharp differences in the test results for slightly different sample periods indicate that the conclusions in Chortareas and Kapetanios (2004) regarding the
(nonlinear) stationarity of the yen RERs around a linear trend are not sustainable, and would be overturned with the inclusion of the data of recent years.

Taking a look at the yen RERs plotted in Figure 1, it can be observed that, for the data of the 70s – 90s, most of these rates seem to fluctuate around a rising trend, reflecting real appreciation of the yen against other currencies during that period. However, a falling trend associated with real yen depreciation appears in the data of the recent decade. A Fourier function in the form of equation (2) is thus estimated to see if it may capture such large rise and fall in the yen RERs. In the selection of the values of \( k \) between 1 and 5, it is found that the residual sum of squares from (2) is minimized when \( \tilde{k} = 1 \) for all the yen RERs in the study. The estimated Fourier functions are plotted in Figure 1 along with the corresponding RERs. They seem to fit well the large swings in the yen RERs.

The FADF and FKSS tests of Christopoulos and León-Ledesma (2010) described in section 2 are then applied to the residual series \( \hat{v}_t \) expressed in (3). The test statistics are displayed in Table 2. The results with a sample of 1974-2006 and 1974-2010 are similar. The null of a unit root is rejected by the FADF and/or FKSS for almost all cases, meaning that the yen RERs appear to be linear or nonlinear stationary around some large temporary mean changes during the post-Bretton Woods period. The only exception is the yen RER of Korea for which the null is not rejected by FADF or FKSS for \( \hat{v}_t \) at the 5% significance level with the sample of 1974-2010. Nevertheless, for this sample period, the same null hypothesis is rejected at the 1% or 5% significance level by the KSS1 and KSS2 statistics in Table 1, implying that the won/yen RER is nonlinear stationary. Moreover, the \( F(\tilde{k}) \) statistics in Table 2 reject the null of linearity at any conventional level of significance.
4. Conclusions

This paper has attempted to shed some light on the puzzling results that the results of applying the conventional or nonlinear unit root tests to the yen RERs in some recent studies appear to be rather sensitive to whether or not including the data of recent years in the studies. It is found that this sensitivity of the test results may come from the failure to take into account the large real appreciations of the yen in earlier years followed by large real depreciations in the recent decade.

The present study utilizes the unit root tests recently developed by Christopoulos and León-Ledesma (2010) that not only allow nonlinearity in the RERs but also account for the presence of multiple smooth temporary breaks in the RERs. The results of the tests show that the large swings in the yen RERs can be well captured by a Fourier function. Consistent with the theory of purchasing power parity, our test results are strongly in support of that the yen RERs in the post-Bretton Woods period can be characterized as being linear or nonlinear stationary around infrequent smooth temporary mean changes. Hence, we conclude that PPP holds in the post-Bretton Woods period for Japan versus other countries in the study even during times of large fluctuations with multiple breaks in RERs.
References


Table 1. Unit root tests on the yen real exchange rates

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<tr>
<td></td>
<td>ADF1</td>
<td>ADF2</td>
<td>KSS1</td>
</tr>
<tr>
<td>Australia</td>
<td>-1.56</td>
<td>-2.85</td>
<td>-2.35</td>
</tr>
<tr>
<td>Canada</td>
<td>-1.64</td>
<td>-3.49*</td>
<td>-1.85</td>
</tr>
<tr>
<td>Germany</td>
<td>-2.24</td>
<td>-3.36</td>
<td>-2.47</td>
</tr>
<tr>
<td>Korea</td>
<td>-1.73</td>
<td>-3.62*</td>
<td>-3.82**</td>
</tr>
<tr>
<td>New Zealand</td>
<td>-2.27</td>
<td>-3.35</td>
<td>-2.74</td>
</tr>
<tr>
<td>UK</td>
<td>-2.70</td>
<td>-3.03</td>
<td>-2.60</td>
</tr>
</tbody>
</table>

Notes: ADF1 and ADF2 are the ADF test statistics without and with a trend, respectively, in the model for testing. The KSS test statistics are referred to KSS1 for de-meaned data, and KSS2 for de-meaned and de-trended data. The 5% and 1% asymptotic critical values for ADF1 are −2.86 and −3.43, respectively, and those for ADF2 are −3.41 and −3.96, respectively. The 5% and 1% asymptotic critical values for KSS1 are −2.93 and −3.48, respectively, and those for KSS2 are −3.40 and −3.93, respectively, taken from Kapetanios et al. (2003, p. 364). * and ** denote rejection of the null of a unit root at the 5% and 1% significance level, respectively.
Table 2. Unit root tests on the yen real exchange rates based on Fourier function

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<tbody>
<tr>
<td></td>
<td>FADF</td>
<td>FKSS</td>
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<tr>
<td>Australia</td>
<td>-3.53</td>
<td>-4.34**</td>
</tr>
<tr>
<td>Canada</td>
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<td>-3.77*</td>
</tr>
<tr>
<td>France</td>
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<td>Germany</td>
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<td>Italy</td>
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<tr>
<td>Korea</td>
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<td>-3.69*</td>
</tr>
<tr>
<td>New Zealand</td>
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</tr>
<tr>
<td>Singapore</td>
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<tr>
<td>UK</td>
<td>-3.51</td>
<td>-4.55**</td>
</tr>
<tr>
<td>US</td>
<td>-4.10*</td>
<td>-3.55</td>
</tr>
</tbody>
</table>

Notes: FADF and FKSS are the test statistics obtained by applying the ADF and KSS tests to the residual series from the regression of RER on a Fourier function. The finite sample critical values are taken from Table 1 and Table 3 in Christopoulos and León-Ledesma (2010). * and ** denote rejection of the null of a unit root at the 5% and 1% significance level, respectively. F(\(\tilde{k}\)) is attained from an F-test with the critical values given in Table 1 of Becker et al. (2006). † indicates rejection of the null of linearity at any conventional significance level.
Figure 1. The yen real exchange rates (——) and the corresponding fourier functions (— — —)