Multinomial Logit Market Share Models: Equilibrium Characteristics and Strategic Implications

Suman Basuroy; Dung Nguyen


Stable URL:
http://links.jstor.org/sici?sici=0025-1909%28199810%2944%3A3C1396%3AMLMSME%3E2.0.CO%3B2-N

Management Science is currently published by INFORMS.

Your use of the JSTOR archive indicates your acceptance of JSTOR’s Terms and Conditions of Use, available at http://www.jstor.org/about/terms.html. JSTOR’s Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at http://www.jstor.org/journals/informs.html.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is an independent not-for-profit organization dedicated to creating and preserving a digital archive of scholarly journals. For more information regarding JSTOR, please contact support@jstor.org.
Multinomial Logit Market Share Models: Equilibrium Characteristics and Strategic Implications

Suman Basu Roy • Dung Nguyen
Faculty of Management, Rutgers University, 180 University Avenue, Newark, New Jersey 07102
Joseph M. Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

We specify and analyze the conditions under which the MNL market share models are appropriate for equilibrium analysis. Our results show that a linear price response function as is often used in empirical research, in conjunction with the typical concavity assumed in a large range of marketing response functions, would yield an interior equilibrium solution. We then consider the optimal reactions on pricing and marketing spending to entry and potential market expansion. In the context of the MNL models, we demonstrate that the entry of a new brand evokes a decrease in the equilibrium prices of the existing brands as a defensive reaction. This is true in both an expanding market as well as a fixed market. However, while new entry into a fixed market would trigger the incumbents to lower the marketing expenditure, we show that firms tend to raise marketing activities as they experience market expansion. Consequently, there exist distinct possibilities that marketing efforts for the existing brands increase in view of entry in an expanding market. Further managerial and marketing implications for endogeneity of the number of firms are explored.

(MNL Market Attraction Models; Equilibrium Analysis; Entry and Market Expansion; Competitive Pricing and Advertising; Free-Entry Equilibrium)

1. Introduction
The large literature on market-share analysis as exemplified in Cooper and Nakanishi's well-known book (1988) has concentrated on justifying and estimating various specifications of market share models such as the linear, the multiplicative, the exponential and in particular, the Multiplicative Competitive Interaction Model (MCI), and the Multinomial Logit Model (MNL). Their book also addresses certain theoretical relations among these models and their estimation implications. The basic justification for some of these specifications is rooted in what has been known as Kotler's fundamental theorem (on market share) and the axiomatic approach in the Bell-Keeney-Little market share theorem (Bell, Keeney, and Little 1975). Since the MCI and the MNL models imply immediately that each brand's market share is nonnegative and the sum of market shares of all brands is unity, both specifications satisfy the so-called logical-consistency requirements (Naert and Bultez 1973). While these two models have not always resulted in superior performance in terms of predictive accuracy in comparison to the other models, it has been argued that they nevertheless yield more meaningful interpretations regarding market-share elasticities and sales-volumes elasticities, notions which managers may find most useful. Consequently, both the MCI and the MNL models have been extensively used in empirical as well as theoretical studies.

What has been missing until recently is the research on the managerial implications of these various market-share models; that is, what would be the normative implications that can be derived on the basis of these com-
peting specifications of market share. In order to address this issue, it would be necessary to formulate models of the firm with certain given market-share specifications. Of particular interest to our present research, we should mention recent efforts along this line of inquiry as reflected in papers published by Karnani (1985) and Grucha, Kumar, and Sudharshan (1992). The specification used in these studies is that of the MCI model. On the other hand, several authors have suggested that the MNL model can be used for equilibrium analyses of marketing decision making (for instance, Carpenter and Lehmann 1985, p. 327; Lilien and Kotler 1983, p. 680; and Lilien and Ruzdics 1982). It appears from the discussion both in Cooper and Nakanishi’s 1988 book and Karnani’s paper that the MCI model is chosen since it follows directly the notion of market attraction, somehow defined. Grucha, Kumar, and Sudharshan (1992) went further by justifying their choice of the MCI on the basis of their belief that the other viable alternative, the MNL, is not appropriate for market equilibrium analyses. It will become clear in the course of our present study that this claim is premature. In fact the MNL represents the market share specification based on which we shall not only analyze the optimal pricing and marketing mix decisions, establish the conditions for equilibrium analyses in a competitive environment, but also explore both post-entry reactions of existing firms in responding to a new competitor as well as their response to potential market expansion.

Our attempt to use the MNL model as the basis for this analysis stems not only from the simple motivation to offer an alternative to the fairly substantial literature on equilibrium analysis using the MCI specification but also from our wish to derive managerial implications for one of the most extensively used market share models in the empirical literature (Gensch and Recker 1979, Guadagni and Little 1983, Gupta 1988, Kamakura and Russell 1989, McFadden 1978, and Punj and Staelin 1978). It is well known that due to its built-in us vs. them structure, this type of attraction models can readily be subject to game theoretic formulations which have proved successful in modeling competitive marketing behavior of the firms. We shall show that the conditions required for such an application are very mild, amounting to no more than typical assumptions of linearity of the price response function and concavity of the market-

2. The MNL Market Share Model and Assumptions

For ease of reference, the main symbols used in this model are defined as follows: \( s_j = \text{brand } j’s \text{ market share} \); \( p_j = \text{price of firm } j’s \text{ product} \); \( n = \text{number of firms} \) in the market; \( \theta = \text{industry specific parameter, reflecting} \) the extent of market expansion; \( m_j = \text{expenditure on marketing activity by firm } j \); \( A_j = \text{attraction of brand } j \); \( D = \text{total industry’s attraction, sum of all the } A_j/s; \)

\[ g(m_j) = \text{a function relating awareness to marketing expenditures, } \]

\[ m_j; \]

\[ f(p_j) = \text{price response function; } FC_j = \text{fixed cost; } MC_j = \text{constant marginal cost; and } \Pi_j = \text{profit of firm } j. \]

Let the attraction of brand \( j \) be specified as:

\[ A_j = e^{\theta(p_j) + g(m_j)}. \]

The market share for brand \( j \) in a market
with \( n \) competitors is given by: 
\[
s_j = A_j / (A_j + \sum_{i \neq j} A_i)
\]
and the profit function of the firm \( j \) can be written as:
\[
\Pi_j = (p_j - MC_j) s_j \times MSZ - m_j - FC_j
\]  
where \( MSZ \) is the market size.

This specification of the MNL market share model represents an aggregate concept, and it has been claimed (see Cooper and Nakanishi 1988, p. 51) that the consumer choice-based utility theory developed by McFadden (1974) and others may serve as a foundation for individual rational decision-making in formulating market shares in the aggregate. Further, the logistic specification has been used recently in the economics literature to explore issues of existence and uniqueness of price equilibrium (Caplin and Nalebuff 1991; Anderson, De Palma, and Thisse 1992) as well as the issue of free entry equilibrium (Besanko, Perry, and Spady 1990).

Now note from the objective function in (1) that the costs that comprise the nominal dollars spent on marketing are subtracted linearly from the revenue (e.g., Kotler 1965, Naert and Weverbergh 1981, and Kumar and Sudharshan 1988). Also, our formulation of the problem differs from the ones that do not consider price as an integral part of the marketing mix (Steckel 1984) and those that assume a fixed profit margin for the firms (Monahan 1987). Most significantly from our present perspective, in several studies the market size has been treated as fixed. In contrast, for the purpose of this study, we will allow the market size to vary. We consider this to be a significant departure from the previous models in two important respects. First, although several markets are mature (hence a major justification for the fixed market assumption), firms have very often been successful in expanding those markets by several means. Second, and more fundamentally, entry typically occurs in expanding markets; hence assuming fixed market size necessarily limits the range of strategic implications which are relevant to managers facing new competition as well as market expansion.

Bell, Keeney, and Little (1975) had suggested that a possible way of constructing the total market size is to let it be a function of the total attraction for all the firms in the market. Karnani (1985) further argued that if one assumes a decreasing marginal utility, it is reasonable to assume that the total market size is a nondecreasing and concave function of the total attraction. Following them and Kotler (1965), we assume a specific functional form for the total market size:
\[
MSZ = MSZ_0 \left( A_j + \sum_{i \neq j} A_i \right)^\theta, \quad 0 < \theta < 1
\]
where \( MSZ_0 \) denotes the base market size, normalized to unity without loss of generality, and \( \theta \) is an industry specific parameter, reflecting the extent of "market expansion." It measures the impact of marketing effort on primary demand for the category at the time the effort is made. Consequently, it can be appropriately termed the "size of primary demand effects." This is so because a firm's marketing activity may generate positive externality for other firms, leading to market expansion. For our present purposes, the parameter \( \theta \) is exogenously given even though a more satisfactory treatment would involve the assumption that market expansion may be endogenous. Finally, the assumption of concavity in the preceding specification requires that \( \theta \) is strictly less than unity.

3. Equilibrium Conditions

The strategy space of the firms consists of the prices and the marketing expenditure \((p_j, m_j)\). Upon noting the exogeneity of \( \theta \) and making the standard assumption that the strategy space is a compact and convex set, existence and uniqueness of a Nash equilibrium to the problem formulated in the previous section can readily be established (Friedman 1977). In other words, the problem facing each firm is:

\[
\max \Pi_j((p_j, m_j)| (p_{-j}, m_{-j}))
\]

where the notation is conventional.

Assuming for convenience, that \( MC_j = 0 \), the Nash equilibrium, \((p_j^*, m_j^*)\), is characterized by the following explicit expressions of the first-order conditions with respect to the price and the marketing expenditure variable:

\[
\frac{\partial \Pi_j}{\partial p_j} = 0 \Rightarrow
\]

\[
p_j \left( \frac{A_j}{D^{1-\theta}} \right) f'(p_j) \left( \sum_{i \neq j} A_i + \theta A_j \right) + \frac{A_j}{D^{1-\theta}} = 0
\]
\[
\frac{\partial \Pi}{\partial m_i} = 0 \Rightarrow p_i \left( \frac{A_i}{D - \theta} \right) \frac{g''(m_i)}{D} \left( \sum_{i \neq j} A_i + \theta A_j \right) = 1. \tag{4}
\]

The sufficient conditions for the extreme points, which satisfy (3) and (4), to be optimal involve second derivatives of profits with respect to price and marketing efforts as well as the cross derivative of these two variables. In anticipation of a later result which shows that the cross derivative is in effect zero in our model, we choose to analyze separately the second derivatives of profit with respect to each of the decision variables in what follows. Further, the manner in which the price variable affects the objective function is rather unique in comparison to all other marketing variables such as advertising and distribution, etc. Finally, as mentioned earlier, there exists a substantial literature that concentrates exclusively on nonpricing marketing activities; hence, analyzing the second derivative of profits with respect to marketing expenditure should yield results directly comparable to those given in the existing literature. To that end, we state the following proposition regarding the nature of the price response function.

(The proof of this proposition, as well as those for other results, is given in the appendix.)

**Proposition 1.** If the price function, \( f(p) \), is such that \( f'(p) < 0 \), and \( f''(p) = 0 \), then the second derivative of profit with respect to price \( \partial^2 \Pi / \partial p^2 \) is negative.

We note that a linear demand specification, among the most common in both empirical and theoretical literature, would satisfy the condition contained in Proposition 1. We shall actually use such a demand specification in our analysis in §5 below. It may be of interest to note that the condition in Proposition 1 is independent of both the market size and the parameter \( \theta \). On the other hand, as we will see next, the condition involving the firm’s marketing expenditure decision is dependent on several parameters. We summarize such a condition in the following proposition:

**Proposition 2.** If the marketing expenditure function is concave and satisfies the following condition:

\[
- \frac{g''(m_i)}{g'(m_i)^2} > \left( \sum_{i \neq j} A_i + \theta A_j \right) D^{-1} \tag{5}
\]

\[
- (1 - \theta) \left( \frac{A_i}{D} \right) \left( \sum_{i \neq j} A_i \right) \left( \sum_{i \neq j} A_i + \theta A_j \right)^{-1}
\]

then the second derivative of profit with respect to the marketing effort variable, \( \partial^2 \Pi / \partial m^2 \), is negative.

The preceding proposition states a condition under which the MNL market share models may be appropriate for equilibrium analysis in a competitive situation and changing market potential. The assumption of concavity in the marketing expenditure response functions implies a positive sign to the expression on the left-hand side of the condition in (5), i.e., \( g''(\cdot) < 0 \). This particular assumption of concavity is well supported in the marketing literature (Blattberg and Golan 1978, Hauser and Gaskin 1984, Tull et al. 1986, Chintagunta 1993, Kotler 1980, Little 1979, Mahajan and Muller 1986, Stern and El-Ansary 1982, and Urban 1974).

The right-hand side of the inequality is a function of the several parameters like \( \theta \), the number of firms, \( n \), etc. Note further that it is strictly positive for admissible values of \( \theta \), and that it is also strictly less than one. Therefore, over and above the conditions of concavity, the marketing expenditure function will have to satisfy additionally the condition stated in Proposition 2. That is, the marketing response function should be sufficiently concave for an interior solution to exist. While this condition appears to involve certain elements of arbitrariness, we can easily find well-known examples in the marketing literature which actually possess properties to satisfy this inequality. We briefly report here a couple of those examples. The first is one of the widely used class of advertising specifications known as the Lanchester models. A generalized model of the Lanchester function (Little 1979, p. 650) for competitive advertising would be: \( g(m_i) = \rho_i m_i / \sum_{j=1}^n \rho_j m_j \), \( j = 1, \ldots, n \) where the coefficients \( \rho_i, i = 1, \ldots, n \) admit differential advertising dollar (\( m_i \)) efficiencies due to media and market characteristics. The condition stated in Proposition 2 holds, since \( -g'(m_i)/g(m_i)^2 = \sum_{j=1}^n \rho_j m_j / \Sigma_{j=1}^n \rho_j m_j \rho_j m_j > 1; j = 1, \ldots, n \).

The second example of the marketing response function is of the form (Carroll, Green, and DeSarbo 1978; and Simon 1982): \( g(m_i) = \log m_i \), from which it follows that the condition stated in Proposition 2 is also satisfied.

We note with particular interest the result that if \( \theta \) is zero, then the condition in Proposition 2 reduces to:

\[
-g''(m_i) / g'(m_i)^2 > \left( \sum_{i \neq j} A_i - A_j \right) / D.
\]
which is, in fact, the condition obtained by Gruca and Sudharsham (1991, p. 481, eq. 10).

If one were to assume in addition that the marketing response function, \( g(\cdot) \) is linear in marketing dollars, then the above condition reduces to:

\[
0 > \frac{1}{D} \left[ \sum_{i=1}^{n} A_i - A_j \right]
\]

which, in effect, requires that firm \( j \)'s market share, \( A_j/D \), exceeds 50% as Gruca and Sudharsham claimed. Clearly, this particular result is based on two rather restrictive assumptions, one of a constant market size and the other of the linearity in the marketing response function.

We now establish the complete second order conditions for an interior Nash solution in both price and marketing expenditure. To formally show those conditions in the context of our model, on the basis of the requirement that the appropriate Hessian matrix \( H_i \) be negative definite for any brand \( j \), we first introduce the following lemma.

**Lemma 1.** The off-diagonal elements of the Hessian matrix, \( H_j \), are zero, that is, the cross-partial derivatives of the profit function with respect to price and marketing expenditure are zero.

The above lemma, coupled with Propositions 1 and 2, yields the following main result regarding the conditions for an interior Nash solution to the problem.

**Proposition 3.** Let the strategy space of price and the marketing expenditure be compact and convex for all firms. Then given the marketing expenditure response function, \( g(\cdot) \), and the price response function, \( f(\cdot) \), sufficient conditions for an interior Nash equilibrium in prices and marketing expenditure are given by Propositions 1 and 2.

In the next section, we analyze managerial and strategic implications regarding the firms' reactions to entry and market expansion.

### 4. Strategic Implications: Entry and Market Expansion

Our examination in the preceding section of conditions under which the MNL models are appropriate for equilibrium analyses enables us to explore managerial and strategic implications using these types of market share models. As mentioned in the introductory section, a major objective of our current study is to derive normative implications for firms under a competitive environment. For our present purposes, the analysis is greatly simplified by focusing only on the symmetric Nash solution to the problem. This simplification, in turn, allows us to effectively address the important issue of potential market expansion that has been largely assumed away in favor of a fixed market size assumption in the literature on this subject. As a result, we will be able to incorporate both the impact of new entry and of market expansion on the existing firms' pricing and other marketing decisions.

The condition of symmetry implies that the attractions of all brands are the same, i.e., \( A_i = A_j = A \) for all \( i, j \), which in turn, implies that the first order conditions for optimality in (3) and (4) takes the following forms:

\[
F(m, n, p, \theta) = p A^g g'(m) \left[ \frac{n - (1 - \theta)}{n^{2 - \theta}} \right] = 1 \quad (6)
\]

\[
G(n, p, \theta) = p f'(p) \left[ \frac{n - (1 - \theta)}{n^{2 - \theta}} \right] + \frac{1}{n^{1 - \theta}} = 0. \quad (7)
\]

Taking the total differentials of \( F \) and \( G \), assuming that \( f'(p) = 0 \) as in Proposition 1, and performing some algebraic manipulations which are described in the appendix yields:

\[
\begin{bmatrix}
\frac{dp}{dm} \\
\frac{dn}{d\theta}
\end{bmatrix} = \begin{bmatrix}
\frac{-pf'k_n + \frac{1 - \theta}{n^{2 - \theta}}}{f'k} \\
\frac{g'k_n}{k(g' + \theta g^2)} + \frac{\ln(n)}{pf'k_n + \frac{1 - \theta}{n^{1 - \theta}}} + g'(1 + \theta p f') \\
\frac{-pf'k_n - \frac{\ln(n)}{n^{1 - \theta}}}{f'k} \\
\frac{-g'(k \ln(A) + k \theta)}{k(g' + \theta g^2)} + \frac{1 - \theta}{pf'k_n + \frac{1 - \theta}{n^{2 - \theta}}} + g'(1 + \theta p f') \\
\end{bmatrix} \begin{bmatrix}
\frac{dp}{dm} \\
\frac{dn}{d\theta}
\end{bmatrix}
\]

(8)
where, for later reference we denote the preceding $2 \times 2$ matrix as $S = [s_{ij}]$ and for notational simplicity, we define: $k(n, \theta) = (n - (1 - \theta)) / n^{2-\theta}$; hence $k_n(n, \theta) = \frac{\partial k(n, \theta)}{\partial n} = [(1 - \theta) / n^{2-\theta}]((2 - \theta) / n) - 1 < 0$ for $n \geq 2$ and $k_\theta(n, \theta) = \frac{\partial k(n, \theta)}{\partial \theta} = (1 + \ln(n)(n - (1 - \theta))) / n^{2-\theta} > 0$ for $n \geq 2$.

The solution summarized in (8) serves as the basis for our analysis in this section. A quick examination of (8) would reveal the potential difficulty in assessing the direction of the impact of entry and market expansion on the existing firms' pricing and marketing expenditure decisions. Fortunately, we are able to determine the direction of those impacts as the following analysis will show. It has been pointed out to us by the editors and the reviewers that the standard interpretation of derivatives of marketing decision variables with respect to the number of firms $n$ as the incumbents' response to entry may not be entirely valid. This is so since entry is essentially a dynamic behavior, profit-driven and thus endogenous. We have, nevertheless, chosen this approach mainly to facilitate comparisons of our results with those currently available in the literature (e.g., Hauser and Shugan 1983; Grucu, Kumar, and Sudharsan 1992; Anderson, De Palma, and Thissle 1992; Bessanko, Perry, and Spady 1990). Further, the impact of increasing competition as reflected through an increase in the number of competitors has been found to yield important general policy and welfare implications for different market structures. On the other hand, we will devote an entire section examining the implications for the endogeneity of the equilibrium number of firms.

### 4.1. Optimal Price Response to Entry

From (8), for a given $\theta$, the effect of entry on price is given as:

$$
\frac{dp}{dn} = s_{11} = -\frac{pk_n(n, \theta)}{k(n, \theta)} + \frac{1 - \theta}{n^{2-\theta}} \frac{1}{k(n, \theta)f'(p)}.
$$

(9)

Using the price elasticity of the representative firm's quantity or share of the total demand, defined as $e = (\partial Q / \partial p) \cdot (p / Q)$ where $Q = A_i / D^{1-\theta}$, the first order condition in (7) for the price variable can be simplified to: $e = k(n, \theta) n^{1-\theta} f'(p) = -1$. Hence, (9) can now be simplified to:

$$
\frac{dp}{dn} = \frac{-pk_n(n, \theta)}{k(n, \theta)} + \frac{1 - \theta}{n} p = \left( -\frac{k_n(n, \theta)}{k(n, \theta)} - \frac{1 - \theta}{n} \right) p.
$$

Therefore, $dp / dn = \left[ - (1 - \theta)(2 - \theta - n) - n(1 - \theta) + (1 - \theta)^2 / n^{2-\theta} k(n, \theta) n \right] p$ where the last equality follows from plugging in the value of $k(n, \theta)$ defined earlier. After some simplification this yields:

$$
\frac{dp}{dn} = -\left( \frac{1 - \theta}{n(n - (1 - \theta))} \right) p < 0
$$

(10)

which proves that the reaction to entry results in the lowering of prices. Hence we can state the following proposition:

**Proposition 4.** Assuming that the condition in Proposition 2 is satisfied, in the context of the MNL market share models, for any of the existing brands in the market, the optimal ex-post defensive pricing strategy in the face of entry under a symmetric equilibrium is to reduce the price.

It is easiest to interpret this result by first considering the case where $\theta = 0$, which implies a fixed market size. From Equation (10) above, we obtain

$$
\frac{dp}{dn} = -\left( \frac{1}{n(n-1)} \right) p < 0.
$$

(11)

Under the condition of a fixed market size, the maximization of profit is actually reduced to the maximization of revenue, since the marginal cost is constant. The first-order condition for the price variable implies that price must be determined in order to yield maximum revenue which would occur at the point of unitary price elasticity, $e = -1$. That is, as $n$ goes up with constant market size, the demand facing each firm is now reduced; to maintain unitary price elasticity of demand the firms would find it optimal to reduce their prices.

The same force continues to prevail in a market environment where the total market expands. In addition, under such a scenario, our result in Proposition 4 shows that a decline in price tends to further increase total attraction, and hence it may enhance profits. In other words, the result indicates that when the market is expanding, new entry will continue to induce existing firms to cut their prices to attain larger profits.

### 4.2. Optimal Marketing Expenditure Response to Entry

In this section we will examine the competitive reactions on marketing efforts in response to entry in the market. First we look at the case of an expanding market. Thereafter we will draw some analogy and com-
parison with the fixed market size case. Before we prove the next result, we prove the following lemma.

**Lemma 2.** If $g(\cdot)$ is concave, then for any $\theta$ between 0 and 1, the following relationship holds:

$$g''(m) + \theta g'(m)^2 < 0. \quad (12)$$

The lemma helps us state and prove the following proposition:

**Proposition 5.** Assuming that the condition in Proposition 2 is satisfied, the optimal ex-post reaction to the existing firms’ entry under a symmetric equilibrium is to reduce marketing activities regardless of the extent of market expansion.

An immediate result of this proposition is given in the following corollary for the case of fixed market size typically assumed in the literature.

**Corollary 1.** With a fixed market size ($\theta = 0$), the optimal defensive reaction to entry under a symmetric equilibrium involves a decrease in the marketing expenditure for all the existing firms.

It is clear that the assumption of concavity regarding the $g$ function yields the result contained in Corollary 1. However, it can be argued that the assumption of a concave $g$ must be made for the fixed market size in equilibrium. Suppose otherwise that $g''$ is positive, (i.e., the marketing response function is convex) then firms will keep increasing marketing activities in order to generate additional profits. As new entry takes place in a fixed market, however, each symmetric firm’s market share is necessarily reduced; increasing marketing activities by all firms will become less effective, implying that decreasing returns will prevail.

In comparing the results obtained regarding the existing firms’ marketing reaction to entry and in view of a forthcoming result below where market expansion tends to raise the firms’ marketing expenditures, Proposition 5 suggests that even in an expanding market, as long as the market is expanding at a steady rate, the incumbents will find it optimal to reduce marketing expenditures in the face of new entry. In practice, one typically observes increasing marketing activities by existing firms as a result of new entry (e.g., the introduction of the 1-800 COLLECT in the long distance telephone market). However, in the general context of the attraction model, marketing activities are not intrinsically designed to deal with possibilities of brand switching. Rather, as the new entrant contributes to the increasing total marketing attraction, it would be optimal reaction for the incumbent to reduce the marketing activity. This particular result should be contrasted with a situation where the market is expanding at different rates, a subject to which we now turn.

### 4.3. Optimal Price Response to Market Expansion

We now investigate the impact of potential market expansion on existing firms’ pricing and marketing decisions. It should be emphasized once again that we are assuming that the extent of market expansion is exogenously determined even though a more challenging model may be based on the assumption of the endogeneity of $\theta$. In this subsection, we examine such an impact of market expansion on the firms’ pricing response while that on their advertising activities will be considered next. The result is summarized in the following proposition.

**Proposition 6.** In the context of the MNL market share models and under a symmetric equilibrium, for a given number of firms in the industry, the faster the market expands, the lower the optimal prices.

Again, the intuition behind this result can be seen by noting the way in which total attraction is translated into sales and, consequently, profits. A price reduction raises firms’ profits simply because at the previous price level, total revenue is still increasing with respect to total sales due to the fast expanding markets. Recall that we have assumed constant marginal cost for simplicity, increasing revenue directly improves profits. We should point out that the assumption of constant marginal cost, while objectionable, allows us to focus on the role of concavity of the marketing response functions on the behavior of the firms. Thus, the result summarized in this proposition should be contrasted to the more standard implications regarding the relationship between increasing output and price reduction, which is driven mainly by improvements in the production processes. In the latter scenario, it is easy to see that as production technology improves, marginal costs decline which in turn leads to reduced prices and subsequently increasing sales. Further, unlike the previous result in Proposition 4 where increasing competition forces firms to cut
their prices, a result that is typical in the literature, our analysis summarized in Proposition 6 suggests that even when the competitive environment facing the firms is unchanged, market expansion alone would induce the firms to cut their prices. This appears to be a particular feature of the attraction-based market share models in general, which we have demonstrated for the specific case of the MNL version.

The software market within the computer industry continues to be one of the fastest growing markets. According to International Data Corp. (IDC), the PC software market has been expanding at 19% per year and is projected to grow at the same rate for the next several years (Standard and Poor’s Industry Surveys, November 24, 1994). In the face of such an expansion, major software vendors have continued to cut prices aggressively. One of the most dramatic examples of such price competition was Computer Associates’ offer in 1993 to deliver its Simply Money financial software package with retail value of $69.95—free. This move was very successful in acquiring an installed base of almost one million users. Thus, price competition in this industry is driven by market expansion and is designed to capture the growing market. Thus in this fast growing market, price on average has been falling about 15% a year (Industry Surveys, October 7, 1993). On the other hand, in slow growing markets, like soaps and dentrifice (growth rates between 2% to 5%), price declines are relatively less, typically far less than 10% (Industry Surveys, January 19, 1995; Robinson 1988).

4.4. Optimal Marketing Expenditure Response to Market Expansion

The optimal reaction of the firms regarding their decision on marketing efforts is summarized as follows.

PROPOSITION 7. In the context of the MNL market share models under a symmetric equilibrium, for a given number of firms in the industry, the faster the market expands, the higher the optimal marketing activities.

Once again, this result is a consequence of the way in which marketing activities, when increased, would enhance total market attraction, leading to increasing sales and hence profits. It is important to recognize that the first-order condition for optimality implies unitary price elasticity which is the driving force behind the firms’ decision to raise marketing expenditures in view of an increase in the value of the parameter θ.

4.5. Responses to Both Entry and Market Expansion

We are now able to combine the previous results to capture the total effect of a concurrent change in both the number of competitors and the potential expansion of the industry. We summarize these results in the following statement.

PROPOSITION 8. In the context of an MNL market share model, under a symmetric equilibrium, the joint effect of n and θ on price is unambiguously negative. The joint effect of n and θ on marketing efforts is ambiguous.

The results contained in Proposition 8 have important implications for further empirical research and managerial decisions. A typical firm operates in an environment that exhibits both the competitiveness asserted by other firms as well as potential for market expansion. Empirical research on the incumbents’ reactions with respect to marketing activities upon entry of new competition explicitly or implicitly takes note of the characteristics of market or industry expansion in the analysis. (see, for instance, Cubbin and Domberger 1988, and Robinson 1988). These empirical studies, while drawing their testing hypotheses from existing theoretical arguments, are not specifically designed on the basis of the theoretical implications of their own, however. On the other hand, theoretical studies on the related subject largely ignore the role of potential market expansion which, as we have argued throughout this paper, is unfortunate, particularly in the attraction-based model formulations. Consequently, efforts to relate empirical findings to certain formal theoretical developments should be viewed to be basically casual.

Within the specific framework we assume in this paper, the results contained in Proposition 8 suggest that firms tend to cut their prices in the face of new competition and opportunities of market expansion. On the other hand, the firms’ reaction regarding marketing efforts is less clear cut. While new competition tends to induce them to cut their marketing efforts to take advantage of the now increased total attraction, the increasing market size requires an increase in marketing expenditures to accommodate it. The net outcome is therefore ambiguous, and thus one should keep in mind these opposite forces in interpreting and constructing
empirical studies using real data which most likely involve both market expansion and competition at the same time.

Our result also contrasts sharply with the existing theoretical results. In the context of the MCI models, Gruca, Kumar, and Sudharshan (1992) conclude that for nondominant brands, the only profit maximizing equilibrium response to entry is to decrease all relevant strategic variables which, in particular, include the marketing expenditure. Their results for nondominant brands, which should be similar to a symmetry equilibrium version of their model, are consistent with earlier studies, in particular the Defender model of Hauser and Shugan (1983). On the contrary, our result summarized in Proposition 8 indicates the distinct possibility that the defensive reaction to entry involves an increase in the marketing efforts by all the incumbents. While this particular theoretical implication may have been revealed in some empirical studies (see Robinson 1988 for some evidence to this effect), we are not aware of a similar result in the theoretical literature for nondominant brands.

5. Free-Entry Equilibrium

In this section we will explore the long-run market equilibrium which endogenizes the number of brands to prevail in the market. In order to obtain closed form solutions on the basis of which the free entry equilibrium can be calculated, we find it necessary to work with a model more specific than those analyzed in the previous section. A challenge for us is to develop a model sufficiently simple to make the mathematics tractable, but at the same time it has to have reasonable empirical support (see, e.g., Nguyen 1997). We specify the following market share model of the MNL form:

$$s_j = \frac{\exp(-\beta p_i + \gamma \ln m_i)}{\sum_{i=1}^n \exp(-\beta p_i + \gamma \ln m_i)}$$ (13)

for firm $j$ based on which its profit function is written as:

$$\Pi_j = p_j s_j - m_j - FC_j$$

where $\beta > 0$ and $\gamma > 0$ are the coefficients reflecting the responses of the market share with respect to price and advertising, respectively. The first order conditions for profit maximization can be shown to yield:

$$p_j = \frac{1}{\beta(1 - s_j)} \cdot m_j = \gamma p_j (s_j - s_j^*)$$

We consider a scenario in which both pricing and promotional efforts are jointly determined. In such a case, we immediately get: $m_j = \gamma / \beta s_j$. Under symmetry, $p_j = p$, and $m_i = m_i = m$; hence,

$$\Pi = \frac{p}{n} - m - FC, \quad m^* = \frac{\gamma}{\beta n}, \quad p^* = \frac{n}{\beta(n - 1)}$$ (14)

from which we obtain:

$$\Pi(n) = \frac{1}{\beta(n - 1)} - \frac{\gamma}{\beta n} - FC$$ (15)

which, upon setting $\Pi(n) = 0$ as a condition for a free-entry equilibrium, yields the following expression regarding the endogenously determined number of firms:

$$n^* = \frac{(\beta FC + (1 - \gamma)) \pm \sqrt{(\beta FC + (1 - \gamma))^2 + 4\beta FC(1 - \gamma)}}{2\beta FC}$$

This expression can be approximated by:

$$n^* \approx 1 + \frac{1 - \gamma}{\beta FC} + \frac{\Delta}{2\beta FC}, \Delta > 0$$ (16)

where it can be verified that in order for $n$ to take on values greater than unity, it is necessary that $\gamma < 1$.

Thus, the inverse relation between the equilibrium number of firms and the fixed costs is established. Therefore, one can rewrite (14) as follows:

$$m^* = \frac{\gamma}{\beta n^*(FC)}, \quad p^* = \frac{n^*(FC)}{\beta(n^*(FC) - 1)}$$ (17)

where we have written explicitly the dependency of $n^*$ on the fixed cost $FC$ on the basis of the relation in (16). We summarize the preceding results in the following statement.

**Proposition 9.** As the fixed costs per firm rise the equilibrium number of firms declines which in turn implies (a) increasing optimal advertising expenditure per firm and (b) increasing optimal price charged by all firms.

The relation in (17) yields the additional result that the equilibrium price varies inversely with the equilibrium number of firms, consistent with an earlier finding.
obtained by Besanko, Perry, and Spady (1990, p. 408). We shall now briefly extend the above results to a scenario where the market is expanding. In particular, we follow the theme developed in the previous section where the extent of market expansion is reflected through the parameter $\theta$.

In an expanding market, parallel expressions to (17) can be obtained:

$$m^*(\theta) = \frac{\gamma}{\beta} D^\theta \frac{1}{n}, \quad p^*(\theta) = \frac{n}{\beta(n - 1 + \theta)}. \quad (17a)$$

The firm's profit function can now be written as:

$$\Pi(n) = \frac{p(\theta)}{n} D^\theta - m(\theta) - FC$$

from which it follows that:

$$\frac{d\Pi}{d\theta} = pA^\theta((\theta - 1)n^{\theta-2}) + n^{\theta-1}A^{\theta} \frac{\partial p}{\partial \theta}$$

$$+ n^{\theta-1}p\theta A^{\theta-1} \frac{\partial m}{\partial \theta}. \quad (18)$$

It can be confirmed readily that, except for the third term in the right-hand side expression of (18), all other three terms are negative. Hence if the market is expanding very fast, that is if $\theta$ is sufficiently large, it may be the case that each existing firm's profits will be increasing with market expansion, which, in turn, attract entry of new firms into the industry. Among the clearest examples of this phenomenon are the spectacular growth of the personal computer industry in sales and in the increasing number of emerging firms in this industry. On the other hand, (18) implies that if an industry experiences relatively slow expansion, even at a positive rate, each incumbent's profits may actually be reduced, leading some firms to leave the market with the consequence of a more concentrated industry equilibrium. Among the most prominent examples of this configuration is the case of the airline industry which has in the last decade or so undergone a substantial transformation in its competitive market structure. As an illustrative example, numerical calculations performed on the basis of the attraction specification in (13) in which $n = 5$, $\beta = 2$, $\gamma = 0.25$ (all within ranges of empirically-supported values), and with an initial market size set at $MSZ_0 = 10,000$ units, indicate that the incumbent's profit would begin to rise with market expansion as $\theta$ reaches a value in the neighborhood of 0.23.

6. Conclusion

In this paper we have specified and analyzed the conditions under which the MNL market share models can form a basis for equilibrium analysis. Our results show that a linear price response function, as is often used in empirical research in conjunction with the typical concavity assumed in a large range of marketing response functions, would yield an interior equilibrium solution. On the basis of these conditions, our paper then deals with the strategic issue of entry in the constructed market and the optimal defensive reactions to entry and potential market expansion. A major feature contained in our analysis is the fact that unlike previous literature, we have allowed the market size to vary. Restricting to zero the industry specific parameter on market expansion yields the fixed market size model typically assumed in the literature. In this manner, at each instance we are able to compare two scenarios—one with the fixed market size and another with a variable market size. We then explore further the implications of the free-entry equilibrium in which the number of firms is endogenously determined.

Our results indicate that in the context of the MNL specification assumed in this paper, the entry of a new brand in the market evokes a decrease in the equilibrium prices of the existing brands as a defensive reaction to entry. This is true in both an expanding market as well as a fixed market. However, while the defensive reaction with respect to the marketing efforts in the face of new entry only is to lower the marketing expenditure, we show that the existing firms tend to raise marketing activities as they experience market expansion. Consequently, there are no clear-cut predictions regarding the firms' marketing expenditures in view of entry in an expanding market.

Our results shed several interesting marketing and managerial insights regarding the defensive reactions to entry in expanding markets. In the personal computer market for example, where potential for expansion is extremely high, the industry has experienced growth in the advertising expenditure as entry occurred. The basic reasoning behind this is that even with entry, the market
is expanding sufficiently fast to yield a larger amount of sales for each brand; each thus increases its marketing efforts to accommodate this potential expansion. In this situation, the positive impact of market expansion on marketing activities dominates the negative impact of entry. However, for a market with low expansion potential, the increase in the number of firms may clutter the market; hence the effect of entry may prevail with the net result of a reduction in the marketing expenditure of the existing brands. Further, on the basis of the theoretical results obtained in this paper as summarized in Proposition 8, it would appear that existing empirical evidence of the incumbents’ marketing reactions to entry, such as those reported in Cubbin and Domberger (1988) and Robinson (1988), may require reinterpretations and additional empirical tests may be constructed to understand fully the significance of the joint impact of entry and market expansion on the firms’ pricing and marketing decisions.\footnote{The authors wish to thank the various readers of earlier drafts of the paper; among them are S. Chan Choi, Sharan Japal, Wagner Kamakura, Rajeev Kohli, Murari Mantra, Muammer Ozier, and Daniel Smith. In particular, they are grateful to the Departmental Editors (Marketing), the Associate Editors, and the reviewers who offer critical and constructive comments on earlier drafts of this paper. The usual disclaimer applies.}

### Appendix

#### PROPOSITION 1

Differentiating the first-order condition in (3) with respect to \( p_j \) gives:

\[
\frac{\partial^2 T_{ij}}{\partial p_j^2} = \left( \frac{A_i}{D^{1-\theta}} \right) \left[ \frac{f'(p_j)}{D} \sum_{i,j} A_i + \theta \frac{f'(p_j)}{D} A_i \right] + \left( \frac{A_i}{D^{1-\theta}} \right) \sum_{i,j} A_i \frac{\partial}{\partial p_i} \left( A_i \right) - \frac{p_j}{p_i} \left( \frac{A_i}{D^{1-\theta}} \right)^2 \left( \frac{f'(p_j)}{D} \right) \frac{f''(p_j)}{f'(p_j)}
\]

which upon considerable simplifications using the first-order condition (3) repeatedly yields:

\[
\frac{\partial^2 T_{ij}}{\partial p_i^2} = -\frac{2}{p_i} \left( \frac{A_i}{D^{1-\theta}} \right) - \left( \frac{A_i}{D^{1-\theta}} \right)^2 \left( \frac{f''(p_j)}{f'(p_j)} \right) f'(p_j) - (1 - \theta) p_i \left( \frac{A_i}{D^{1-\theta}} \right) \frac{f'(p_j)^2 A_i}{D^2} \sum_{i,j} A_i.
\]

The above expression shows that if the price function is such that \( f''(p) = 0 \), then

\[
\frac{\partial^2 T_{ij}}{\partial p_j^2} = -\frac{2}{p_i} \left( \frac{A_i}{D^{1-\theta}} \right) - (1 - \theta) p_i \left( \frac{A_i}{D^{1-\theta}} \right) \frac{f'(p_j)^2 A_i}{D^2} \sum_{i,j} A_i < 0.
\]

#### PROOF (PROPOSITION 2)

Again, differentiating the first-order condition in (4) with respect to the marketing expenditure variable, \( m_i \) yields:

\[
\frac{\partial^2 T_{ij}}{\partial m_i^2} = p_i \left( \frac{g'(m_i)}{D} \sum_{i,j} A_i + \theta \frac{g'(m_i)}{D} A_i \right) \frac{\partial}{\partial m_i} \left( A_i \right) + \frac{A_i}{D^{1-\theta}} \sum_{i,j} A_i \frac{\partial}{\partial m_i} \left( \frac{g'(m_i)}{D} \right) + \theta \frac{g'(m_i)}{D} A_i.
\]

After substantial simplifications of these terms, applying the first-order condition repeatedly, we derive the following:

\[
\frac{\partial^2 T_{ij}}{\partial m_i^2} = \frac{g'(m_i)}{D} \left[ \sum_{i,j} A_i + \theta A_i \right] + \frac{g''(m_i)}{g'(m_i)} \left[ \sum_{i,j} A_i + \theta A_i \right] - (1 - \theta) \left[ \frac{D}{\sum_{i,j} A_i + \theta A_i} \right] \frac{g'(m_i)}{D} \left( \frac{A_i}{D^{1-\theta}} \right \sum_{i,j} A_i \right] \]

which would be negative if \(-g'(m_i)/g''(m_i) > ((\sum_{i,j} A_i + \theta A_i)/D) - (1 - \theta)(A_i/D)(\sum_{i,j} A_i + \theta A_i)/((\sum_{i,j} A_i + \theta A_i))\) as claimed.

#### PROOF (LEMMA 1)

On the basis of Equation (3), the cross-partial derivative is given by:

\[
\frac{\partial}{\partial m_i} \left( \frac{\partial T_{ij}}{\partial p_j} \right) = \left( \frac{A_i}{D^{1-\theta}} \right) p_i \frac{f''(p_j)}{D} \left[ \frac{\partial}{\partial m_i} \left( \sum_{i,j} A_i + \theta A_i \right) D^{-1} \right] + p_i \frac{f''(p_j)}{D} \left[ \frac{\partial}{\partial m_i} \left( \sum_{i,j} A_i + \theta A_i \right) D^{-1} + 1 \right] \frac{A_i}{D^{1-\theta}}.
\]

After simplifying and using the first-order condition in (3), we have:

\[
\left( \frac{A_i}{D^{1-\theta}} \right) \left[ p_i \frac{f''(p_j)}{D} \left( \sum_{i,j} A_i + \theta A_i \right) D^{-1} + 1 \right] \frac{A_i}{D^{1-\theta}} = 0.
\]

This completes the proof of the lemma.

#### Derivation of Equation (8)

Taking the total differential of \( F \) in (6), we obtain:

\[
dF = \left[ A \varphi'(m)k(n, \theta) + p \theta A \varphi'(p) \varphi'(m) \varphi(n, \theta) \right] dp + \left[ p A \varphi'(m)k(n, \theta) + p \varphi'(m) \theta A \varphi'(n, \theta) \right] dm + [p A \varphi'(m)k(n, \theta) \varphi(n, \theta)] d\theta.
\]

Similarly, the total differential of \( G \) in (7) yields:

1406 MANAGEMENT SCIENCE / Vol. 44, No. 10, October 1998
\[ dG = \left[ (f'(p) + pf'(p))k(n, \theta)dp \right] + \left[ pf'(p)k(n, \theta) - \frac{1 - \theta}{n^{1-\theta}} \right] dn + \left[ pf'(p)k(n, \theta) + \frac{\ln n}{n^{1-\theta}} \right] d\theta = 0. \]

The preceding equations for the \( dF \) and \( dG \) yield the following matrix relationship:

\[ \begin{bmatrix} A^k(n)[g'(m) + p\theta f'(p)] & pA^k(n)[g'(m) + \theta g'(m)^2] \\ k(n)f'(p) + pf'(p) \end{bmatrix} \begin{bmatrix} dp \\ dn \end{bmatrix} = 0 = \begin{bmatrix} -pA^k(g'(m))k(n, \theta) - pg'(m)k(n, \theta)A^\theta \ln A - pA^k\theta g'(m)k(n, \theta) \\ -pf'(p)k(n, \theta) - \frac{1 - \theta}{n^{1-\theta}} - pf'(p)k(n, \theta) - \frac{\ln n}{n^{1-\theta}} \end{bmatrix} \begin{bmatrix} dn \\ d\theta \end{bmatrix}. \]

which can be rewritten in a compact form as

\[ \mathbf{B}\begin{bmatrix} dp \\ dn \end{bmatrix} = \mathbf{C}\begin{bmatrix} dn \\ d\theta \end{bmatrix}, \]

where matrices \( \mathbf{B} \) and \( \mathbf{C} \) are defined accordingly.

The preceding equation would yield the solution of \( dp \) and \( dm \) in terms of \( dn \) and \( d\theta \):

\[ \begin{bmatrix} dp \\ dm \end{bmatrix} = (\mathbf{B}^{-1} \cdot \mathbf{C})\begin{bmatrix} dn \\ d\theta \end{bmatrix}. \]

which will be, for convenience, partitioned as:

\[ \begin{bmatrix} dp \\ dm \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}\begin{bmatrix} dn \\ d\theta \end{bmatrix}, \]

from which Equation (8) obtains by routine calculation.

PROOF. (LEMMA 2) If the condition in (5) is satisfied then, under symmetry, it implies that

\[ -\frac{g'(m)}{g'(m)^2} > \frac{n - 1 + \theta}{n} - \frac{1 - \theta}{n} \frac{n - 1}{n + 1} = h(n, \theta). \quad (5a) \]

However, one can establish that \( [\partial h(n, \theta)/\partial n] > 0 \). Further, for \( n = 1, h(1, \theta) = 0 \). Therefore, for \( n > 1, h(n, \theta) > 0 \). Thus, by the inequality condition in (5a), for any \( n > 1 \), we get

\[ -\frac{g'(m)}{g'(m)^2} > h(n, \theta) > \theta \]

which completes the proof of the lemma. □

PROOF. (PROPOSITION 5) From (8), taking into account the first order condition that \( \varepsilon = -1 \), and after some simplification, we obtain:

\[ \frac{dm}{dn} = \left( -1 - \frac{\theta}{k(n, \theta)n^{1-\theta}} \right) \left( \frac{\ln(n)g'(m)}{k(n, \theta) + \theta g'(m)^2} \right) - \left( k(n, \theta)n(g'(m) + \theta g'(m)^2) \right). \]

Noting Lemma 2, and the fact that \( g'(m) > 0 \), the preceding relation yields:

\[ \text{Sign}(\frac{dm}{dn}) = \text{Sign}(\left( -1 - \frac{\theta}{k(n, \theta)n^{1-\theta}} \right)) \]

where the last expression follows since \( k(n, \theta) + (n - 1 - \theta)/n^{1-\theta} \).

Noting further that \( pf'(p) = -n/(n - (1 - \theta)) \) we have

\[ \text{Sign}(\frac{dm}{dn}) = \text{Sign}\left( \frac{\ln(n)}{(n - (1 - \theta))^2} - 1 \right) < 0, \]

for \( 0 < \theta < 1, n \approx 2. \] □

PROOF. (PROPOSITION 6) From (8), we have:

\[ \frac{dp}{d\theta} = s_{12} = -\frac{pA^k(n, \theta)}{k(n, \theta)} - \frac{\ln(n)}{n^{1-\theta} k(n, \theta) f'(p)} - \frac{\theta}{k(n, \theta) n^{1-\theta}} \]

due to the first order condition, since \( \varepsilon = k(n, \theta)n^{1-\theta} pf'(p) = -1 \). It follows that \( dp/d\theta < 0. \) □

PROOF. (PROPOSITION 7) From (8), we have:

\[ \frac{dm}{d\theta} = s_{22} = \left( -\frac{g'(m) (\ln(A) + \ln(n))}{k(n, \theta)} \right) \]

\[ + \frac{\theta g'(m)}{k(n, \theta)} \left( \frac{pf'(p)k(n, \theta) + \ln(n)}{n^{1-\theta}} \right) \left( g'(m) + \theta g'(m)^2 \right)^{-1} \]

where \( k(n, \theta) pf'(p) = -1/n^{1-\theta} \). Using Lemma 2 once again, it follows that:

\[ \text{Sign}(\frac{dm}{d\theta}) = \text{Sign}(\left( \frac{\ln(n)g'(m)}{k(n, \theta)n^{1-\theta}} \right)) \]

\[ = \text{Sign}(\left( 1 - \frac{\theta}{k(n, \theta)n^{1-\theta}} \right)) \text{ since } n > 1, g'(m) > 0. \]

Since \( k(n, \theta)n^{1-\theta} = [n - (1 - \theta)]/n \), we have \( \text{Sign}(dm/d\theta) = \text{Sign}(n - 1)(1 - \theta) > 0, n > 1, 0 < \theta < 1. \)

PROOF. (PROPOSITION 8) The proposition follows directly from (8) where

\[ dp = s_{11}dn + s_{12}d\theta, \quad dm = s_{21}dn + s_{22}d\theta. \]

Upon applying Propositions 4, 5, 6, and 7, we get the desired result. □
References


Accepted by Dipak C. Jain; received June 28, 1995. This paper has been with the authors for 3 revisions.